DESIGN OF MACHINE ELEMENTS – I

Subject Code :18ME54 **IA Marks :** 20
Hours/Week :05 **Exam Hours :** 03
Total Hours :50 **Exam Marks :** 80

Module-1

Fundamentals of Mechanical Engineering Design Mechanical engineering design, Phases of design process, Design considerations, Engineering Materials and their Mechanical properties, Standards and Codes, Factor of safety, Material selection.

Static Stresses: Static loads .Normal, Bending, Shear and Combined stresses. Stress concentration

and determination of stress concentration factor. 10 Hours

Module -2

Design for Impact and Fatigue Loads Impact stress due to Axial, Bending and Torsional loads. Fatigue failure: Endurance limit, S-N Diagram, Low cycle fatigue, High cycle fatigue, modifying factors: size effect, surface effect. Stress concentration effects, Notch sensitivity, fluctuating stresses, Goodman and Soderberg relationship, stresses due to combined loading, cumulative fatigue damage. 10 Hours

Module -3

Design of Shafts, Joints, Couplings and Keys Torsion of shafts, design for strength and rigidity with steady loading, ASME codes for power transmission shafting, shafts under combined loads.

Design of Cotter and Knuckle joints, Rigid and flexible couplings, Flange coupling, Bush and Pin type coupling and Oldham‟s coupling. Design of keys-square, saddle, flat and father. 10 Hours

Module - 4

Riveted Joints and Weld Joints Rivet types, rivet materials, failures of riveted joints, Joint Efficiency, Boiler Joints, Lozanze Joints, Riveted Brackets, eccentrically loaded joints.

Types of welded joints, Strength of butt and fillet welds, welded brackets with transverse and

parallel fillet welds, eccentrically loaded welded joints. 10 Hours

Module - 5

Threaded Fasteners and Power Screws Stresses in threaded fasteners, Effect of initial tension, Design of threaded fasteners under static loads, Design of eccentrically loaded bolted joints.

Types of power screws, efficiency and self-locking, Design of power screw, Design of screw jack: (Complete Design). 10 Hours

Text Books:

1. Design of Machine Elements, V.B. Bhandari, Tata McGraw Hill Publishing Company Ltd., New Delhi, 2nd Edition 2007.

2. Mechanical Engineering Design, Joseph E Shigley and Charles R. Mischke. McGraw Hill International edition, 6th Edition, 2009.

Design Data Handbook:

1. Design Data Hand Book, K. Lingaiah, McGraw Hill, 2nd Ed.
2. Data Hand Book, K. Mahadevan and Balaveera Reddy, CBS Publication
3. Design Data Hand Book, S C Pilli and H. G. Patil, I. K. International Publisher, 2010.

Reference Books:

1. Machine Design, Robert L. Norton, Pearson Education Asia, 2001.
2. Engineering Design, George E. Dieter, Linda C Schmidt, McGraw Hill Education, Indian Edition, 2013.
3. Design of Machined Elements, S C Pilli and H. G. Patil, I. K. International Publisher, 2017.
4. Machine Design, Hall, Holowenko, Laughlin (Schaum‟s Outline series) adapted by S.K Somani, tata McGraw Hill Publishing company Ltd., New Delhi, Special Indian Edition, 2008
5. Design of Machine Elewments -1, Dr.M.H.Annaiah, Dr. C.N. Chandrappa, Dr.J.Suresh Kumar, New Age International Publishers.

CONTENTS

UNIT 1

INTRODUCTION

Instructional Objectives

- Explain what is design?
- Describe the machine and its designer,
- Illustrate the procedure of design,
- Know materials used in mechanical design, and
- Understand the considerations for manufacturing.

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

If the end product of the engineering design can be termed as mechanical then this may be termed as Mechanical Engineering Design. Mechanical Engineering Design may be defined as: **"Mechanical Engineering Design is defined as iterative decision making process to describe a machine or mechanical system to perform specific function with maximum economy and efficiency by using scientific principles, technical information, and imagination of the designer."** A designer uses principles of basic engineering sciences, such as Physics, Mathematics, Statics, Dynamics, Thermal Sciences, Heat Transfer, Vibration etc.

Classifications of Machine Design:

The machine design may be classified as follows:

1. ***Adaptive design.*** In most cases, the designer‟s work is concerned with adaptation

of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

2. ***Development design.*** This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. ***New design.*** This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design. The designs, depending upon the methods used, may be classified as follows:

(a) ***Rational design.*** This type of design depends upon mathematical formulae of principle of mechanics.

(b) ***Empirical design.*** This type of design depends upon empirical formulae based on the practice and past experience.

(c) ***Industrial design.*** This type of design depends upon the production aspects to manufacture any machine component in the industry.

(d) ***Optimum design.*** It is the best design for the given objective function under the specified constraints. It may be achieved by minimizing the undesirable effects.

(e) ***System design.*** It is the design of any complex mechanical system like a motor car.

(f) ***Element design.*** It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(g) ***Computer aided design.*** This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimization of a design.

General Considerations in Machine Design

Following are the general considerations in designing a machine component:

1. ***Type of load and stresses caused by the load.*** The load, on a machine component, may act in several ways due to which the internal stresses are set up. The various

types of load and stresses are discussed later.

2. *Motion of the parts or kinematics of the machine.* The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required.

The motion of the parts may be : (*a*) Rectilinear motion which includes unidirectional and reciprocating motions. (*b*) Curvilinear motion which includes rotary, oscillatory and simple harmonic. (*c*) Constant velocity. (*d*) Constant or variable acceleration.

3. *Selection of materials.* It is essential that a designer should have a thorough knowledge of the properties of the materials and their behaviour under working conditions. Some of the important characteristics of materials are: strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc. The various types of engineering materials and their properties are discussed later.

4. *Form and size of the parts.* The form and size are based on judgment. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.

5. *Frictional resistance and lubrication.* There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

6. *Convenient and economical features.* In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible. The economical operation of a machine which is to be used for production or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. ***Use of standard parts.*** The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order. The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers and taps and also to decrease the number of wrenches required.

8. ***Safety of operation.*** Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with operation of the machine.

9. ***Workshop facilities.*** A design engineer should be familiar with the limitations of this employer‟s workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.

10. ***Number of machines to be manufactured.*** The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for small number of the product will not permit any undue expense in the workshop processes, so that the designer should restrict his specification to standard parts as much as possible.

11. ***Cost of construction.*** The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of handmade samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions should be to reduce the manufacturing cost to the minimum.

12. ***Assembling.*** Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

Manufacturing considerations in Machine design

Manufacturing Processes

The knowledge of manufacturing processes is of great importance for a design engineer. The following are the various manufacturing processes used in Mechanical Engineering.

1. ***Primary shaping processes.*** The processes used for the preliminary shaping of the machine component are known as primary shaping processes. The common operations used for this process are casting, forging, extruding, rolling, drawing, bending, shearing, spinning, powder metal forming, squeezing, etc.

2. ***Machining processes.*** The processes used for giving final shape to the machine component, according to planned dimensions are known as machining processes. The common operations used for this process are turning, planning, shaping, drilling, boring, reaming, sawing, broaching, milling, grinding, hobbing, etc.

3. ***Surface finishing processes.*** The processes used to provide a good surface finish for the machine component are known as surface finishing processes. The common operations used for this process are polishing, buffing, honing, lapping, abrasive belt grinding, barrel tumbling, electroplating, super finishing, sheradizing, etc.

4. ***Joining processes***. The processes used for joining machine components are known

as joining processes. The common operations used for this process are welding, riveting, soldering, brazing, screw fastening, pressing, sintering, etc.

5. ***Processes effecting change in properties.*** These processes are used to impart certain specific properties to the machine components so as to make them suitable for particular operations or uses. Such processes are heat treatment, hot-working, cold-working and shot peening.

General Procedure in Machine Design

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows:

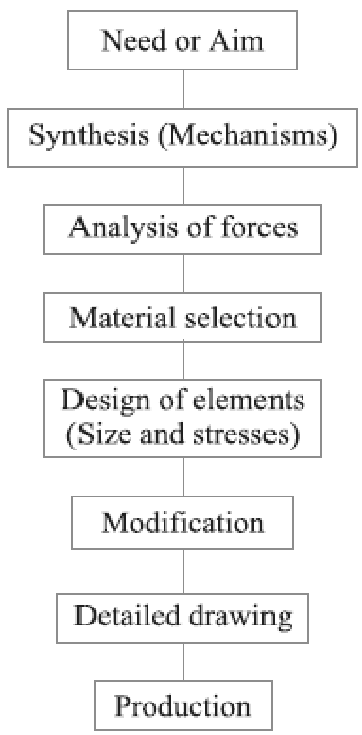

Fig.1. General Machine Design Procedure

1. ***Recognition of need.*** First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. ***Synthesis*** **(*Mechanisms*).** Select the possible mechanism or group of mechanisms which will give the desired motion.

3. ***Analysis of forces.*** Find the forces acting on each member of the machine and the energy transmitted by each member.

4. ***Material selection.*** Select the material best suited for each member of the machine.

5. *Design of elements* (*Size and Stresses*). Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. *Modification.* Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. *Detailed drawing.* Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. *Production.* The component, as per the drawing, is manufactured in the workshop. The flow chart for the general procedure in machine design is shown in Fig.1.

Standards and Standardization

Standards in Design:

Standard is a set of specifications, defined by a certain body or an organization, to which various characteristics of a component, a system, or a product should conform. The characteristics may include: dimensions, shapes, tolerances, surface finish etc.

Types of Standards Used In Machine Design:

Based on the defining bodies or organization, the standards used in the machine design can be divided into following three categories:

(i) Company Standards: These standards are defined or set by a company or a group of companies for their use.

(ii) National Standards: These standards are defined or set by a national apex body and are normally followed throughout the country. Like BIS, AWS.

(iii) International Standards: These standards are defined or set by international apex body and are normally followed throughout the world. Like ISO, IBWM.

Advantages:

- Reducing duplication of effort or overlap and combining resources
- Bridging of technology gaps and transferring technology
- Reducing conflict in regulations

- Facilitating commerce
- Stabilizing existing markets and allowing development of new markets
- Protecting from litigation

B.I.S DESIGNATIONS OF THE PLAIN CARBON STEEL:

Plain carbon steel is designated according to BIS as follows:

1. The first one or two digits indicate the 100 times of the average percentage content of carbon.
2. Followed by letter “C”
3. Followed by digits indicates 10 times the average percentage content of Manganese “Mn”.

B.I.S DESIGNATIONS OF ALLOY STEEL:

Alloy carbon steel is designated according to BIS as follows:

1. The first one or two digits indicate the 100 times of the average percentage content of carbon.

2. Followed by the chemical symbol of chief alloying element.

3. Followed by the rounded off the average percentage content of alloying element as per international standards.

4. Followed by the chemical symbol of alloying elements followed by their average percentage content rounded off as per international standards in the descending order.

5. If the average percentage content of any alloying element is less than 1%, it should be written with the digits up to two decimal places and underlined.

Engineering materials and their properties:

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials. Now, we shall discuss the commonly used engineering materials and their properties in Machine Design.

Classification of Engineering Materials

The engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminum, etc.

2. Non-metals, such as glass, rubber, plastic, etc.

The metals may be further classified as:

(a) Ferrous metals and (*b*) Non-ferrous metals.

The ****ferrous metals*** are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

The ***non-ferrous*** metals are those which have a metal other than iron as their main constituent, such as copper, aluminum, brass, tin, zinc, etc.

Selection of Materials for Engineering Purposes

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost. The following factors should be considered while selecting the material:

1. Availability of the materials,

2. Suitability of the materials for the working conditions in service, and

3. The cost of the materials.

The important properties, which determine the utility of the material, are physical, chemical and mechanical properties. We shall now discuss the physical and mechanical properties of the material in the following articles.

Physical Properties of Metals

The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity, and melting point. The following table shows the important physical properties of some pure metals.

Mechanical Properties of Metals

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

1. Strength. It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.

2. **Stiffness.** It is the ability of a material to resist deformation under stress. The modulus

of elasticity is the measure of stiffness.

3. Elasticity. It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

4. Plasticity. It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. Ductility. It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. Brittleness. It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.

7. Malleability. It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.

8. Toughness. It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable in parts subjected to shock and impact loads.

9. Machinability. It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.

10. Resilience. It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

11. Creep. When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called **creep.** This property is considered in designing internal combustion engines, boilers and turbines.

12. Fatigue. When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as ***fatigue**. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

13. Hardness. It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are dependent on the method of making the test. The hardness of a metal may be determined by the following tests:

(a) Brinell hardness test,

(b) Rockwell hardness test,

(c) Vickers hardness (also called Diamond Pyramid) test, and

(d) Shore scleroscope.

Stress

When some external system of forces or loads acts on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as ***unit stress*** or simply a ***stress*.** It is denoted by a Greek letter sigma (ζ). Mathematically,

$$\text{Stress, } \zeta = P/A$$

Where P = Force or load acting on a body, and
A = Cross-sectional area of the body.

In S.I. units, the stress is usually expressed in Pascal (Pa) such that 1 Pa = 1 N/m^2. In actual practice, we use bigger units of stress *i.e.* megapascal (MPa) and gigapascal (GPa), such that

Strain

1 $a = 1 \times 10^9$ N/m^2 = 1 kN/mm^2

When a system of forces or loads act on a body, it undergoes some

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deformation. This deformation per unit length is known as ***unit strain*** or simply a ***strain.*** It is denoted by a Greek letter epsilon (ε). Mathematically,

$$\text{Strain, } \varepsilon = \delta l / l \text{ or } \delta l = \varepsilon . l$$

Where δl = Change in length of the body,

l= Original length of the body.

Tensile Stress and Strain

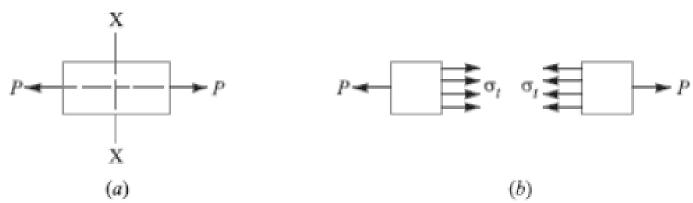


Fig. Tensile stress and strain

When a body is subjected to two equal and opposite axial pulls *P* (also called tensile load) as shown in Fig. (*a*), then the stress induced at any section of the body is known as ***tensile stress*** as shown in Fig. (*b*). A little consideration will show that due to the tensile load, there will be a decrease in cross-sectional area and an increase in length of the body. The ratio of the increase in length to the original length is known as ***tensile strain.***

Let P = Axial tensile force acting on the body,
A = Cross-sectional area of the body,
l = Original length, and δl= Increase in length.

Then, Tensile stress, $\zeta_t = P/A$ and tensile strain, $\varepsilon_t = \delta l / l$

Young's Modulus or Modulus of Elasticity

Hooke's law* states that when a material is loaded within elastic limit, the stress is directly proportional to strain, *i.e* . $\sigma \propto \varepsilon$ or $\sigma = E.\varepsilon$

$$E = \frac{\sigma}{\varepsilon} = \frac{P \times l}{A \times \delta l}$$

where *E* is a constant of proportionality known as ***Young's modulus*** or ***modulus of elasticity.*** In S.I. units, it is usually expressed in GPa *i.e*. GN/m^2 or kN/mm^2. It may be noted that Hooke's law holds good for tension as well as compression.

The following table shows the values of modulus of elasticity or Young's modulus (*E*) for the materials commonly used in engineering practice.

Values of „E" for the commonly used engineering materials.

Material	Modulus of elasticity (E) GPa
Steel and Nickel	200 to 220
Wrought iron	190 to 200
Cast iron	100 to 160
Copper	90 to 110
Brass	80 to 90
Aluminium	60 to 80
Timber	10

Stress-Strain Curves

Properties are quantitative measure of materials behavior and mechanical properties pertain to material behaviors under load. The load itself can be **static** or **dynamic**. A gradually applied load is regarded as static. Load applied by a universal testing machine upon a specimen is closet example of gradually applied load and the results of tension test from such machines are the basis of defining mechanical properties. The dynamic load is not a gradually applied load – then how is it applied. Let us consider a load *P* acting at the center of a beam, which is simply supported at its ends. The reader will feel happy to find the stress (its maximum value) or deflection or both by using a formula from Strength of Materials. But remember that when the formula was derived certain assumptions were made. One of them was that the load *P* is gradually applied. Such load means that whole of *P* does not act on the beam at a time but applied in instalments. The instalment may be, say *P*/100 and thus after the 100^{th} instalment is applied the load *P* will be said to be acting on the beam. If the whole of *P* is placed upon the beam, then it comes under the category of the dynamic load, often referred to as **Suddenly Applied Load**. If the load *P* falls from a height then it is a **shock load**. A fatigue load is one which changes with time. Static and dynamic loads can remain unchanged with time after first application or may alter with time (increase or reduce) in which case, they are fatigue load. A load which remains constantly applied over a long time is called creep load.

All Strength of Material formulae are derived for static loads. Fortunately the stress caused by a suddenly applied load or shock load can be correlated with the stress caused by gradually applied load. We will invoke such relationships as and when needed. Like stress formulae, the mechanical properties are also defined and determined under gradually applied loads because such determination is easy to control and hence economic. The properties so determined are influenced by sample geometry and size, shape and surface condition, testing machines and even operator. So the properties are likely to vary from one machine to another and from one laboratory to another. However, the static properties carry much less influence as compared to dynamic (particularly fatigue) properties. The designer must be fully aware of such influences because most machines are under dynamic loading and static loading may only be a dream.

It is imperative at this stage to distinguish between **elastic constants** and mechanical properties. The elastic constants are dependent upon type of material and not upon the sample. However, strain rate (or rate of loading) and temperature may affect elastic constants. The materials used in machines are basically **isotropic** (or so assumed) for which two independent elastic constants exist whereas three constants are often used in correlating stress and strains. The three constants are Modules of Elasticity (E), Modulus of Rigidity (G) and Poisson‟s Ratio (v). Any one constant can be expressed in terms of other two.

An isotropic material will have same value of E and G in all direction but a natural material like wood may have different values of E and G along fibres and transverse to fibre. Wood is non-isotropic. Most commonly used materials like iron, steel, copper and its alloys, aluminum and its alloys are very closely isotropic while wood and plastic are non-isotropic. The strength of material formulae are derived for isotropic materials only.

The leading mechanical properties used in design are ultimate tensile strength, yield strength, percent elongation, hardness, impact strength and fatigue strength. Before we begin to define them, we will find that considering tension test is the most appropriate beginning.

Tension Test

The tension test is commonest of all tests. It is used to determine many mechanical properties. A cylindrical machined specimen is rigidly held in two jaws of universal testing machine. One jaw is part of a fixed cross-head, while other joins to the part of moving cross-head. The moving cross-heads moves slowly, applying a gradually applied load upon the specimen.

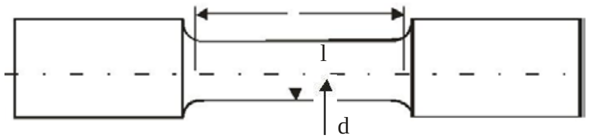


Figure 1.1 : Tension Test Specimen

The specimen is shown in Figure 1.1. The diameter of the specimen bears constant ratio with the gauge length which is shown is Figure 1.1 as distance between two gauge points marked at the ends of uniform diameter length. In a standard specimen $\lambda = 5$. The diameter, *d*, and gauge length, *l*, are measured before the specimen is placed in the machine. As the axial force increases upon the specimen, its length increases, almost imperceptibly in the beginning. But if loading continues the length begins to increase perceptibly and at certain point reduction in diameter becomes visible, followed by great reduction in diameter in the local region of the length. In this localized region the two parts of the specimen appear to be separating as the machine continues to operate but the load upon the specimen begins to reduce. Finally at some lesser load the specimen breaks, with a sound, into two pieces. However, the increase in length and reduction of load may not be seen in all the materials. Specimens of some materials show too much of extension and some show too little. The reader must be conversant with the elastic deformation, which is recoverable and plastic deformation, which is irrecoverable. Both type of deformations occur during the test. The appearance of visible decrease in the diameter in the short portion of length (called necking) occurs when the load on the specimen is highest. The machines of this type have arrangement (devices) for the measurement of axial force, *P*, and increase in length, δ. The values of force, *P* and extensions, δ can be plotted on a graph. Many machines have *x*-*y* recorder attached and direct output of graph is obtained. The stress is denoted by σ and calculated as P/A where, *A* is the original area of cross-section. Although the area of cross-section of specimen begins to change as the deformations goes plastic, this reduction is seen at and after the maximum load. The separation or fracture into two pieces can be seen to have occurred on smaller diameter. Yet, the stress all through the test, from beginning to end, is represented by $\sigma = P/A$. The strain is defined as the ratio of change in length at any load *P* and original length *l* and represented by ε, i.e. $\varepsilon = \delta/l$ at all loads. Since *A* and *l* are constants hence nature of graph between *P* and δ

(load-extension) or between σ and ε (stress-strain) will be same. Figure 1.2 shows a stress-strain diagram, typically for a material, which has extended much before fracture occurred.

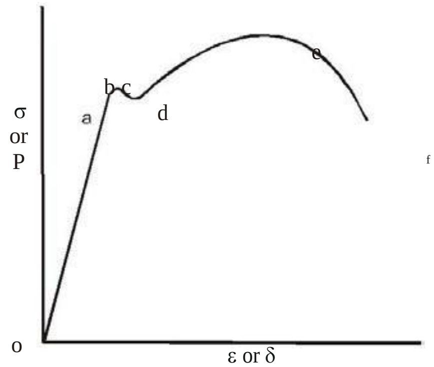


Figure 1.2 : Typical σ – ε Diagram

At first we simply observe what this diagram shows. In this diagram *o* is the starting point and *oa* is straight line. Along line *oa*, stress (σ) is directly proportional to strain (ε). Point *b* indicates the elastic limit, which means that if specimen is unloaded from any point between *o* and *b* (both inclusive) the unloading curve will truly retrace the loading curve. Behaviour of specimen material from point *b* to *c* is not elastic. In many materials all three points of *a*, *b* and *c* may coincide. At *c* the specimen shows deformation without any increase in load (or stress). In some materials (notably mild or low carbon steel) the load (or stress) may reduce perceptibly at *c*, followed by considerable deformation at the reduced constant stress. This will be shown in following section. However, in most materials *cd* may be a small (or very small) region and then stress starts increasing as if the material has gained strength. Of course the curve is more inclined toward ε axis. This increase in stress from *d* to *e* is due to strain hardening. Also note again that *ob* is elastic deformation zone and beyond *b* the deformation is elastic and plastic – meaning that it is part recoverable and part irrecoverable. As the deformation increases plastic deformation increases while elastic deformation remains constant equal to that at *b*. If the specimen is unloaded from any point in the plastic deformation region the unloading curve will be parallel to elastic deformation curve as shown in Figure 1.3.

e

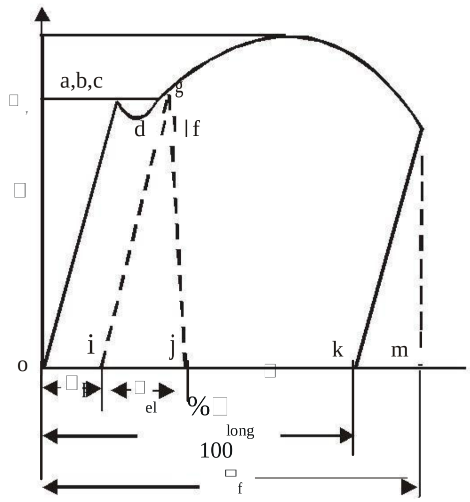


Figure 1.3 : σ – ε Diagram for a Ductile Material

Percent Elongation

From any point *g* the unloading will be along *gi* where *gi* is parallel to *oa*. *oi* is the strain which remains in the specimen or the specimen is permanently elongated by $l\ \varepsilon_p$. The total strain at *g* when the specimen is loaded is $oj = \varepsilon_p + \varepsilon_{el}$ where ε_{el} is recoverable part. Atfracture, i.e. at point *f*, if one is able to control and unload the specimen just before fracture, the unloading will follow *f k*. The strain *ok* is an important property because deformation is defined as percent elongation. Hence, *ok* = % elongation/100. Percent elongation is important property and is often measured by placing two broken pieces together and measuring the distance between the gauge points. You can easily see that after the fracture has occurred, the specimen is no more under load, hence elastic deformation (which is equal to km) is completely recovered. However, in a so-called ductile material *km* << *om*. If the distance between gauge points measured on two broken halves placed together is l_f, then

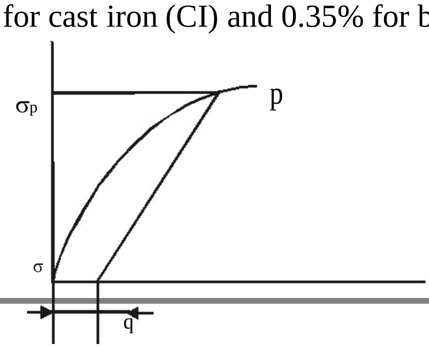
$$\% \text{ Elongation} = \frac{l_f - l}{l} \times 100$$

The gauge length has pronounced effect on % elongation. Since the major amount of deformation occurs locally, i.e. over very small length smaller gauge length will result in higher % elongation. After l/d > 5 the % elongation becomes independent of gauge length. % elongation is an indication of very important property of the material called **ductility**. The ductility is defined as the property by virtue of which a material can be drawn into wires which means length can be increased and diameter can be reduced without fracture. However, a ductile material deforms plastically before it fails. The property opposite to ductility is called **brittleness**. A brittle material does not show enough plastic deformation. Brittle materials are weak under tensile stress, though they are stronger than most ductile materials in compression

Ultimate Tensile Strength, Yield Strength and Proof Stress

The maximum stress reached in a tension test is defined as **ultimate tensile strength**. As shown in Figure 1.3 the highest stress is at point *e* and ultimate tensile stress (UTS) is represented by σ_u. Some authors represent it by S_u. The point *c* marks the beginning while *d* marks the end of yielding. *c* is called upper yield point while *d* is called the lower yield point. The stress corresponding to lower yield point is defined as the **yield strength**. For the purposes of machines, the part has practically failed if stress reaches yield strength, (σ_Y), for this marks the beginning of the plastic deformation. Plastic deformation in machine parts is not permissible. Hence one may be inclined to treat σ_Y as failure criterion. We will further discuss this later in the unit.

It is unfortunate to note that many practical materials show $\sigma - \varepsilon$ diagrams which do not have such well defined yielding as in Figures 1.2 and 1.3. Instead they show a continuous transition from elastic to plastic deformation. In such cases yield strength (σ_Y) becomes difficult to determine. For this reason an alternative, called **proof stress**, is defined which is a stress corresponding to certain predefined strain. The proof stress is denoted by σ_p. A $\sigma - \varepsilon$ diagram for a material, which shows no distinct yield is shown in Figure 1.5. The proof stress is determined corresponding to proof strain ε_p which is often called offset. By laying ε_p on strain axis to obtain a point *q* on ε axis and drawing a line parallel to elastic line to cut the $\sigma - \varepsilon$ curve at *p* the proof stress σ_p is defined. Then σ_p is measured on stress axis. The values of proof strain or offset have been standardized for different materials by American Society for Testing and Materials (*ASTM*). For example, offset for aluminum alloys is 0.2%, same is for steels while it is 0.05% for cast iron (CI) and 0.35% for brass and bronze



ε_p ε

Figure 1.5: Proof Stress (σ_p) Corresponding to Offset ε_p

Toughness and Resilience

Since the force, which pulls the tension test specimen, causes movement of its point of application, the work is done. This work is stored in the specimen and can be measured as energy stored in the specimen. It can be measured as area under the curve between load (P) and elongation (Δl). In case of $\sigma - \varepsilon$ curve area under the curve represents energy per unit volume.

Toughness is regarded as ability of a material to absorb strain energy during elastic and plastic deformation. The resilience is same capacity within elastic range. The maximum toughness will apparently be at fracture, which is the area under entire $\sigma - \varepsilon$ diagram. This energy is called **modulus of toughness**. Likewise the maximum energy absorbed in the specimen within elastic limit is called **modulus of resilience**. This is the energy absorbed in the tension specimen when the deformation has reached point *a* in Figure 1.2. But since in most materials the proportional limit, elastic limit (points *a* and *b* in Figures 1.2 and 1.3) seem to coincide with yield stress as shows in Figure 1.3, the modules of resilience is the area of triangle as shown in Figure 1.6.

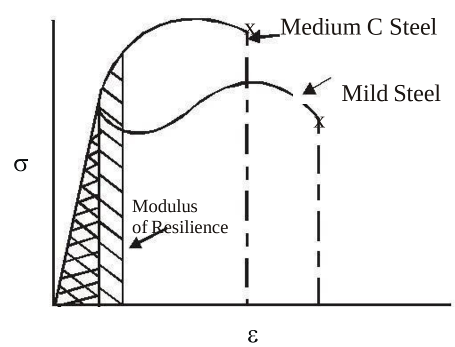


Figure 1.6 : Resilience and Toughness for Two Materials

It can be seen that modulus of resilience is greater for medium carbon steel than for mild steel, whereas modulus of toughness of two materials may be closely same. Medium carbon steel apparently has higher UTS and YS but smaller percent elongation with respect to mild steel. High modulus of resilience is preferred for such machine parts, which are required to store energy. Springs are good example. Hence, springs are made in high yield strength materials.

Stress Strain Diagram for Mild Steel

Mild steel as steel classification is no more a popular term. It was in earlier days that group of steel used for structural purposes was called mild steel. Its carbon content is low and a larger group of steel, named low carbon steel, is now used for the same purposes. We will read about steel classification later. Mild steel was perhaps developed first out of all steels and it was manufactured from Bessemer process by blowing out carbon from iron in a Bessemer converter. It was made from pig iron. The interesting point to note is that this steel was first studied through $\sigma - \varepsilon$ diagram and most properties were studied with respect to this material.

The term **yield strength** (YS) is frequently used whereas yield behavior is not detectable in most steel varieties used today. It is mild steel, which very clearly shows yield behavior and upper and lower, yield points. Figure 1.7 shows a typical $\sigma - \varepsilon$ diagram for mild steel.

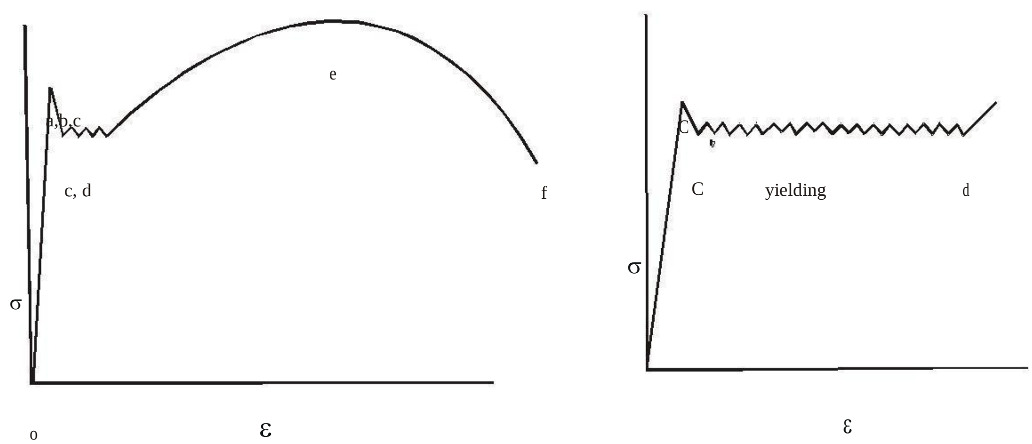


Figure 1.7 : $\sigma - \varepsilon$ Diagram for Mild Steel

The proportional limit, elastic limit and upper yield point almost coincide. *d* is lower yield point and deformation from c' to *d* is at almost constant stress level. There is perceptible drop in stress from *c* to c'. The deformation from c' to *d* is almost 10 times the deformation upto *c*. It can be seen effectively if strain is plotted on larger scale, as shown on right hand side in Figure 1.7, in which the ε scale has been doubled.

The mechanism of yielding is well understood and it is attributed to line defects, dislocations.

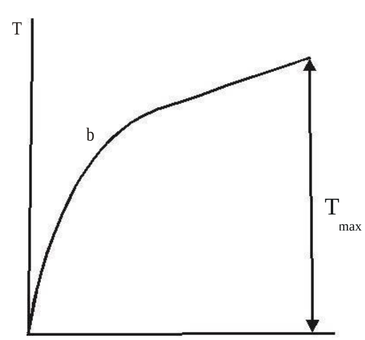
The UTS normally increases with increasing strain rate and decreases with increasing temperature. Similar trend is shown by yield strength, particularly in low carbon steel.

Compression Strength

Compression test is often performed upon materials. The compression test on ductile material reveals little as no failure is obtained. Brittle material in compression shows specific fracture failure, failing along a plane making an angel greater than 45^o with horizontal plane on which compressive load is applied. The load at which fracture occurs divided by area of *X*-section is called compressive strength. For brittle material the stress-strain curves are similar in tension and compression and for such brittle materials as CI and concrete modulus of elasticity in compression are slightly higher than that in tension.

Torsional Shear Strength

Another important test performed on steel and CI is *torsion test*. In this test one end of specimen is rigidly held while twisting moment or torque is applied at the other end. The result of test is plotted as a curve between torque (T) and angle of twist or angular displacement θ. The test terminates at fracture. The T – θ curves of a ductile material is very much similar to load extension or σ – ε curve of tensile test except that the torque does not reduce after attaining a maximum value but fracture occurs at maximum torque. It is because of the fact that there is no reduction in the sectional area of the specimen during the plastic deformation. The elastic limit in this case can be found as the point where straight line terminates and strain hardening begins, marked as point *b* in Figure 1.8. Mild steel will show a marked yielding while other ductile materials show a smooth transition from elastic to plastic deformation. The plastic deformation zone in torsion is much larger than in torsion because the plastic deformation beginning from outer surface and spreads inside while in tension the stress being uniform over the *X*-section the plastic deformation spreads over entire section at the same time.



O θ

Figure 1.8 : Torque-twist Diagram in Torsion

The *modulus of rapture or ultimate torsional shear strength* is calculated form

$$\tau_u = \frac{3}{4} \frac{T_{max}}{J} \frac{d}{2}$$

where T_{max} is maximum torque, *j* is polar moment of inertia of the specimen section of diameter *d*. From the *T* diagram the slope of linear region can be found as proportional to modulus of rigidity, which is ratio of shearing stress to shearing strain.

Elastic Constants

Within elastic limit the stress is directly proportional to strain. This is the statement of Hooke‟s law and is true for direct (tensile or compressive) stress and strain as well as for shearing (including torsional shearing) stress and strain. The ratio of direct stress to direct strain is defined as *modulus of elasticity* (*E*) and the ratio of shearing stress and shearing strain is defined as *modulus of rigidity* (*G*). Both the modulus is called elastic constants. For isotropic material *E* and *G* are related with Poisson‟s ratio

$$G = \frac{E}{2\,(1\ v)}$$

Poisson‟s ratio which is the ratio of transverse to longitudinal strains (only magnitude) in tensile test specimen is yet another elastic constant. If stress σ acts in three directions at a point it is called volumetric stress and produces volumetric strain. The ratio of volumetric stress to volumetric strain according to Hooke‟s law is a constant, called *bulk modulus* and denoted by *K*. It is important to remember that out of four elastic constants, for an isotropic material only two are independent and other two are dependent. Thus *K* can also be expressed as function of any two constants.

$$K = \frac{E}{3\,(1 \square 2v)}$$

It may be understood that elastic constants *E* and *G* are not determined from tension or torsion test because the machines for these tests undergo adjustment of clearance and also some deformation, which is reflected in diagram ordinarily. The constants are determined from such devices, which show large deformation for comparatively smaller load. For example, *E* is determined by measuring deflection of a beam under a central load and *G* is determined by measuring deflection of a close-coiled helical spring an axial load. Poisson‟s ratio is normally not measured directly but is calculated from above equation.

Shear Stress and Strain

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called ***shear stress.***

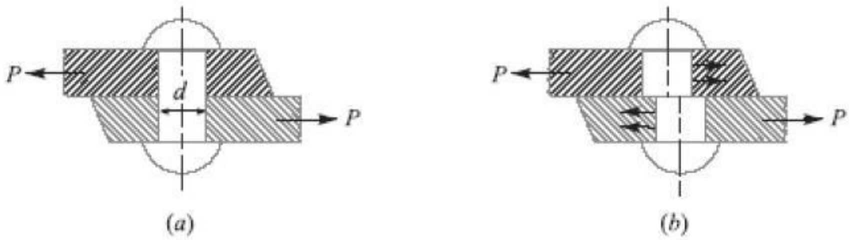


Fig. Single shearing of a riveted joint.

The corresponding strain is known as ***shear strain*** and it is measured by the angular deformation accompanying the shear stress. The shear stress and shear strain are denoted by the Greek letters tau (τ) and phi (φ) respectively. Mathematically,

$$\text{Shear stress, } \tau = \frac{\text{Tangential force}}{\text{Resisting area}}$$

Consider a body consisting of two plates connected by a rivet as shown in Fig. (*a*). In this case, the tangential force *P* tends to shear off the rivet at one cross-section as shown in Fig. (*b*). It may be noted that when the tangential force is resisted by one cross-section of the rivet (or when shearing takes place at one cross-section of the rivet), then the rivets are said to be in ***single shear.*** In such a case, the area resisting the shear off the rivet,

$$A = \frac{\pi}{4} \times d^2$$

And shear stress on the rivet cross-section

$$\tau = \frac{P}{A} = \frac{P}{\frac{\pi}{4} \times d^2} = \frac{4P}{\pi d^2}$$

Now let us consider two plates connected by the two cover plates as shown in Fig. *(a).* In this case, the tangential force *P* tends to shear off the rivet at two cross-sections as shown in Fig.

rivet (or when the shearing takes place at Two cross-sections of the rivet), then the rivets are said to be in *double shear*. In such a case, the area resisting the shear off the rivet,

$$A = 2 \times \frac{\pi}{4} \times d^2 \qquad \text{(For double shear)}$$

and shear stress on the rivet cross-section.

$$\tau = \frac{P}{A} = \frac{P}{2 \times \frac{\pi}{4} \times d^2} = \frac{2P}{\pi d^2}$$

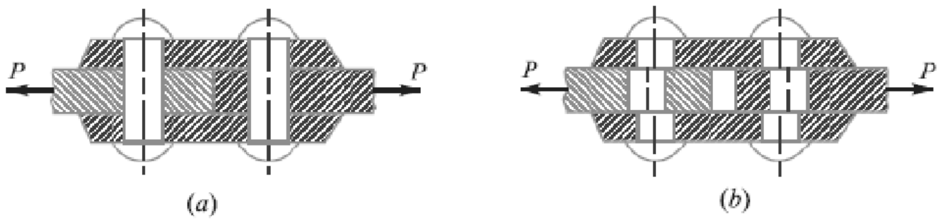


Fig. Double shearing of a riveted joint.

Notes:

1. All lap joints and single cover butt joints are in single shear, while the butt joints with double cover plates are in double shear.

2. In case of shear, the area involved is parallel to the external force applied.

3. When the holes are to be punched or drilled in the metal plates, then the tools used to perform the operations must overcome the ultimate shearing resistance of the material to be cut. If a hole of diameter '*d*' is to be punched in a metal plate of thickness '*t*', then the area to be sheared,

$$A = \pi d \times t$$

And the maximum shear resistance of the tool or the force required to punch a hole,

$$P = A \times \tau_u = \pi d \times t \times \tau_u$$

Where σ_u= Ultimate shear strength of the material of the plate.

Shear Modulus or Modulus of Rigidity

It has been found experimentally that within the elastic limit, the shear stress is directly proportional to shear strain. Mathematically

$$\tau \propto \phi \qquad \text{or} \quad \tau = C \,.\, \phi \quad \text{or} \quad \tau / \phi = C$$

Where, η= Shear stress,

θ = Shear strain, and

C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by *N* or *G.*

The following table shows the values of modulus of rigidity (*C*) for the materials in everyday use:

Values of C for the commonly used materials

Material	*Modulus of rigidity (C) GPa*
Steel	80 to 100
Wrought iron	80 to 90
Cast iron	40 to 50
Copper Brass	30 to 50
Timber	30 to 50
	10

Linear and Lateral Strain

Consider a circular bar of diameter *d* and length *l*, subjected to a tensile force *P* as shown in Fig. (*a*).

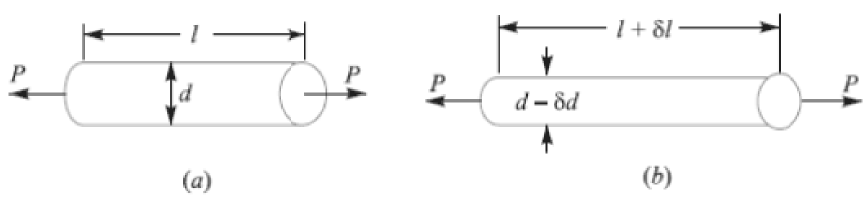


Fig. Linear and lateral strain.

A little consideration will show that due to tensile force, the length of the bar increases by an amount δ*l* and the diameter decreases by an amount δ*d*, as shown in Fig. (*b*). similarly, if the bar is subjected to a compressive force, the length of bar will decrease which will be followed by increase in diameter.

It is thus obvious, that every direct stress is accompanied by a strain in its own direction which is known as ***linear strain*** and an opposite kind of strain in every direction, at right angles to it, is known as ***lateral strain.***

Poisson's Ratio

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain, Mathematically,

$$\frac{\text{LateralStrain}}{\text{LinearStrain}} = \text{Constant}$$

This constant is known as **Poisson's ratio** and is denoted by 1/*m* or μ.

Following are the values of Poisson's ratio for some of the materials commonly used in engineering practice.

Values of Poisson"s ratio for commonly used materials

S.No.	*Material*	*Poisson 's ratio (1/m or μ)*
1	**Steel** Cast	0.25 **to** 0.33
2	iron Copper	0.23 **to** 0.27
3	Brass	0.31 to 0.34
4	Aluminium	0.32 **to** 0.42
5	Concrete	0.32 to 0.36
6	Rubber	0.08 to 0.18

Volumetric Strain

When a body is subjected to a system of forces, it undergoes some changes in its dimensions. In other words, the volume of the body is changed. The ratio of the change in volume to the original volume is known as ***volumetric strain.*** Mathematically, volumetric strain,

$$\varepsilon_v = \delta V / V$$

Where δV = Change in volume, and V = Original volume

Notes : 1. Volumetric strain of a rectangular body subjected to an axial force is given as

$$\varepsilon_v = \frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right); \text{ where } \varepsilon = \text{Linear strain.}$$

2. Volumetric strain of a rectangular body subjected to three mutually perpendicular forces is given by $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$. Where, ε_x, ε_y and ε_z are the strains in the directions *x*-axis, *y*-axis and *z*-axis respectively.

Bulk Modulus

When a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as ***bulk modulus.*** It is usually denoted by *K*. Mathematically, bulk modulus,

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\delta V / V}$$

Relation between Young"s Modulus and Modulus of Rigidity

The Young's modulus (*E*) and modulus of rigidity (*G*) are related by the following relation,

$$G = \frac{m.E}{2\,(m+1)} = \frac{E}{2\,(1+\mu)}$$

Principal Stresses and Principal Planes

In the previous chapter, we have discussed about the direct tensile and compressive stress as well as simple shear. Also we have always referred the stress in a plane which is at right angles to the line of action of the force. But it has been observed that at any point in a strained material, there are three planes, mutually perpendicular to each other which carry direct stresses only and no shear stress. It may be noted that out of these three direct stresses, one will be maximum and the other will be minimum. These perpendicular planes which have no shear stress are known as principal planes and the direct stresses along these planes are known as principal stresses. The planes on which the maximum shear known as planes of maximum shear.

Determination of Principal Stresses for a Member Subjected to Bi-axial Stress

When a member is subjected to bi-axial stress (i.e. direct stress in two mutually perpendicular planes accompanied by a simple shear stress), then the normal and shear stresses are obtained as discussed below:

Consider a rectangular body ABCD of uniform cross-sectional area and unit thickness subjected to normal stresses ζ1 and ζ2 as shown in Fig. (a). In addition to these normal stresses, a shear stress η also acts. It has been shown in books on „Strength of Materials" that the normal stress across any oblique section such as EF inclined at an angle θ with the direction of ζ2, as shown in Fig. (a), is given by

$$\sigma_t = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 + \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta \qquad ...(i)$$

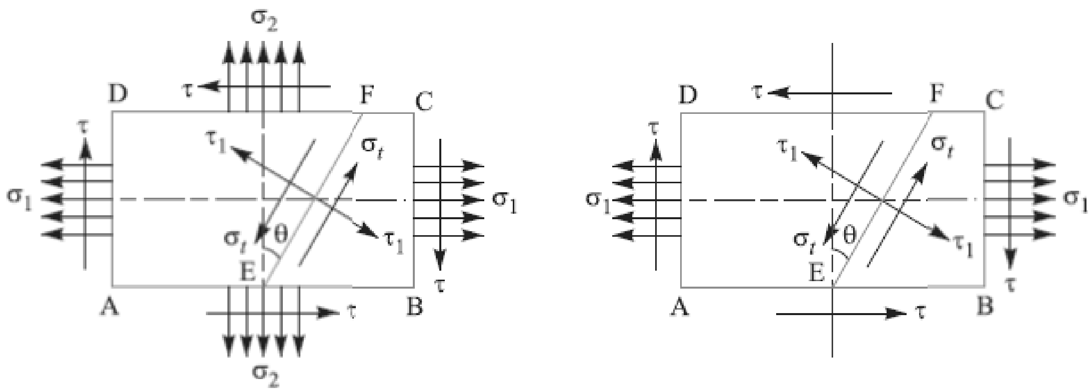
And tangential stress (i.e. shear stress) across the section EF,

Since the planes of maximum and minimum normal stress (i.e. principal planes) have no shear stress, therefore the inclination of principal planes is obtained by equating η1 = 0 in the above equation (ii), i.e.

$$\tau_1 = \frac{1}{2}(\sigma_1 - \sigma_2)\sin 2\theta - \tau \cos 2\theta \qquad ...(ii)$$

$$\frac{1}{2}(\sigma_1 - \sigma_2)\sin 2\theta - \tau \cos 2\theta = 0$$

$$\tan 2\theta = \frac{2\,\tau}{\sigma_1 - \sigma_2} \qquad ...(iii)$$



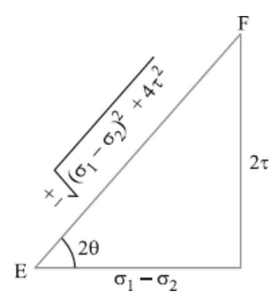
(*a*) Direct stress in two mutually prependicular planes accompanied by a simple shear stress.

(*b*) Direct stress in one plane accompanied by a simple shear stress.

Fig. Principal stresses for a member subjected to bi-axial stress

We know that there are two principal planes at right angles to each other. Let θ_1 and θ_2 be the inclinations of these planes with the normal cross-section. From the following Fig., we find that

$$\sin 2\theta = \pm \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$



$$\therefore \qquad \sin 2\theta_1 = + \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\text{and} \qquad \sin 2\theta_2 = - \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\text{Also} \qquad \cos 2\theta = \pm \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\therefore \qquad \cos 2\theta_1 = + \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\text{and} \qquad \cos 2\theta_2 = - \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

The maximum and minimum principal stresses may now be obtained by substituting the values of sin 2θ and cos 2θ in equation (i).

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}$$

So, Maximum principal (or normal) stress, and minimum principal (or noral) stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}$$

The planes of maximum shear stress are at right angles to each other and are inclined at 45° to the principal planes. The maximum shear stress is given by one-half the algebraic difference between the principal stresses, i.e.

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}$$

Notes: 1. when a member is subjected to direct stress in one plane accompanied by a simple shear stress, then the principal stresses are obtained by substituting ζ2 = 0 in above equations.

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

2. In the above expression of ζt2, the value of $\frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$ is more than ζ1/2 Therefore the nature of ζt2 will be opposite to that of ζt1, i.e. if ζt1 is tensile then ζt2 will be compressive and vice-versa.

Application of Principal Stresses in Designing Machine Members

There are many cases in practice, in which machine members are subjected to combined stresses due to simultaneous action of either tensile or compressive stresses combined with shear stresses. In many shafts such as propeller shafts, C-frames etc., there are direct tensile or compressive stresses due to the external force and shear stress due to torsion, which acts normal to direct tensile or compressive stresses. The shafts like crank shafts, are subjected simultaneously to torsion and bending. In such cases, the maximum principal stresses, due to the combination of tensile or compressive stresses with shear stresses may be obtained. The results obtained in the previous article may be written as follows:

1. Maximum tensile stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

2. Maximum compressive stress,

$$\sigma_{c(max)} = \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right]$$

3. Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

Where, ζt = Tensile stress due to direct load and bending,

ζc = Compressive stress, and

η = Shear stress due to torsion.

Notes: 1. When η = 0 as in the case of thin cylindrical shell subjected in pressure, then ζtmax = ζt.

2. When the shaft is subjected to an axial load (P) in addition to bending and twisting moments as in the propeller shafts of ship and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (ζb). This will give the resultant tensile stress or compressive stress (ζt or ζc) depending upon the type of axial load (i.e. pull or push).

Problems:

1. A point in a structural member subjected to plane stress is shown in Fig.Q.l(b). Determine the following:

i) Normal and tangential stress intensities on plane MN inclined at 45°.

ii) Principal stresses and their direction

iii) Maximum shear stress and the direction of the planes on which it occurs.

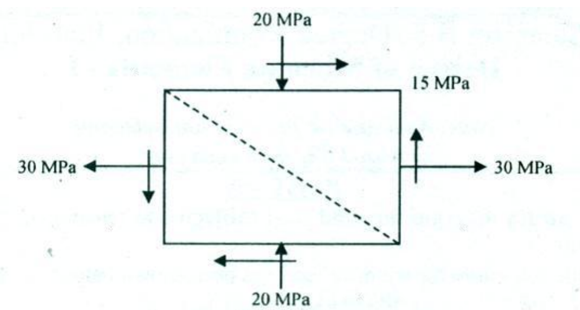


Fig. Q 1(b)

σ_x =30Mpa σ_y=20Mpa τ_{xy} =15Mpa Ø = 45°

(i) Normal stress on plane MN (– from HB)

$$\sigma_\phi = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi$$

$$= \frac{30+20}{2}\sin 2\times 45 + 15\cos(2\times 45)$$

=–25 N/mm2

Negative sign indicates τ_ϕ tends to produce CW rotation

(ii) Maximum principal stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{30-20}{2} + \sqrt{\left(\frac{30+20}{2}\right)^2 + 15^2} = 34.15N/mm^2 \text{ (Tensile)}$$

Minumim Principal stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau x^2 y}$$

= – 24.155 N/mm2 (compressive)

Location :

Angles at which the principal stresses act

$$\phi_{1,2} = \frac{1}{2}\tan^{-1}\left[\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right] \quad \text{where } \phi_1 \text{ \& } \phi_2 \text{ are } 90^\circ \text{ apart}$$

$$= \frac{1}{2}\tan^{-1}\left[\frac{2\times 15}{30+20}\right]$$

$\phi_1 = 15.48$ & $\phi_2 = 105.48$

(iii) Maximum shear stress

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

$$= \pm\sqrt{\left(\frac{30-20}{2}\right)^2 + 15^2} = \pm 29.15 N/mm^2$$

Location:

Angles at which maximum shear stress act

$\phi_{13} = \phi_1 + 45^\circ$

$= 15.48 + 45 = 60.48$

$\phi_{12} = \phi_1 + 135^\circ$

$= 15.48 + 135 = 150.48$

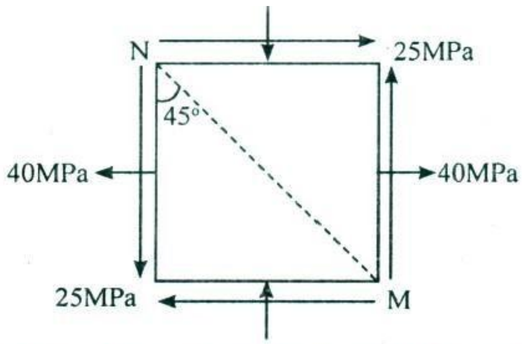
2. Point in a structural member is subjected to plane state of stree as shown in Fig. Ql(b). Determine the following:

i) Normal and tangential stress intensities at a angle of e = 45°

ii) Principal stresses 0', and 0'2 and their directions.

iii) Maximum shear stress and its plane.

Solution:



Given, $\sigma_x = 40\text{MPa}$, $\sigma_z = -30\text{MPa}$, $\sigma_{xy} = 25\text{MPa}$, $\theta = 45°$

(i) Normal stress on plane MN,

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{40-30}{2} + \frac{40+30}{2}\cos(2\times 45) + 25\sin(2\times 45)$$

$$= 30\text{N}/\text{mm}^2$$

Shear stress on plane MN,

$$\tau_\theta = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{40+30}{2}\sin(2\times 45) + 25\cos(2\times 45)$$

$$= -35\text{N}/\text{mm}^2$$

(ii) Maximum principal stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{40-30}{2} + \sqrt{\left(\frac{40+30}{2}\right)^2 + 25^2}$$

$$= 48.0116\text{N}/\text{mm}^2\ (\text{Tensile})$$

Minimum principal stress

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{40-30}{2} - \sqrt{\left(\frac{40+30}{2}\right)^2 + 25^2}$$

$$= -38.0116\text{N}/\text{mm}^2\ (\text{Tensile})$$

Location :

Angles at which principal stress acts

$$\theta_{1,2} = \frac{1}{2}\text{Tan}^{-1}\left[\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right]$$

$$= \frac{1}{2}\text{Tan}^{-1}\left[\frac{2\times 25}{40+30}\right]$$

$$\theta_1 = 17.769^\circ$$

$$\theta_2 = 90 + 17.769^\circ = 107.769^\circ$$

(iii) Maximum shear stress,

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{40+30}{2}\right)^2 + 25^2}$$

$$= \pm 43.0116\text{N}/\text{mm}^2$$

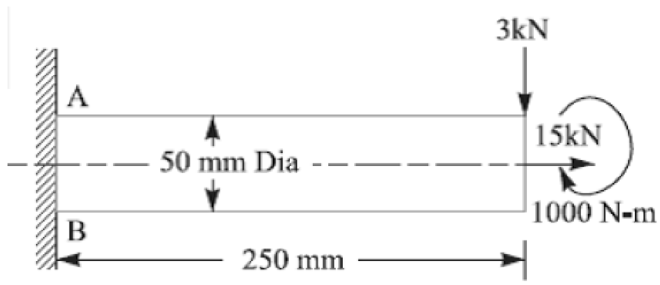
Location :

Angles at which maximum shear stress acts :

$$\theta_1^2 = \theta_1 + 45^\circ = 17.769 + 45 = 62.769^\circ$$

$$\theta_2^1 = \theta_1 + 135^\circ = 17.769 + 135 = 152.769^\circ$$

3. A shaft, as shown in Fig., is subjected to a bending load of 3 kN, pure torque of 1000Nm and an axial pulling force of 15 kN stresses. Calculate the stresses at A and B.



Solution. Given : $W = 3$ kN $= 3000$ N ; $T = 1000$ N-m $= 1 \times 10^6$ N-mm ; $P = 15$ kN $= 15 \times 10^3$ N ; $d = 50$ mm; $x = 250$ mm

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2$$

$$= \frac{\pi}{4} (50)^2 = 1964 \text{ mm}^2$$

$\therefore$ Tensile stress due to axial pulling at points A and B,

$$\sigma_o = \frac{P}{A} = \frac{15 \times 10^3}{1964} = 7.64 \text{ N/mm}^2 = 7.64 \text{ MPa}$$

Bending moment at points A and B,

$$M = W.x = 3000 \times 250 = 750 \times 10^3 \text{ N-mm}$$

Section modulus for the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32} (50)^3$$

$$= 12.27 \times 10^3 \text{ mm}^3$$

$\therefore$ Bending stress at points A and B,

$$\sigma_b = \frac{M}{Z} = \frac{750 \times 10^3}{12.27 \times 10^3}$$

$$= 61.1 \text{ N/mm}^2 = 61.1 \text{ MPa}$$

This bending stress is tensile at point A and compressive at point B.

∴ Resultant tensile stress at point *A*,

$$\sigma_A = \sigma_b + \sigma_o = 61.1 + 7.64$$
$$= 68.74 \text{ MPa}$$

and resultant compressive stress at point *B*,

$\sigma_B - \sigma_b \quad \sigma_o - 61.1 \quad 7.64 - 53.46$ MPa

We know that the shear stress at points *A* and *B* due to the torque transmitted,

$$\tau = \frac{16\,T}{\pi\,d^3} = \frac{16 \times 1 \times 10^6}{\pi\,(50)^3} = 40.74 \text{ N/mm}^2 = 40.74 \text{ MPa} \qquad \ldots\left(\because\ T = \frac{\pi}{16} \times \tau \times d^3\right)$$

Stresses at point A

We know that maximum principal (or normal) stress at point *A*,

$$\sigma_{A(max)} = \frac{\sigma_A}{2} + \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right]$$
$$= \frac{68.74}{2} + \frac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$
$$= 34.37 + 53.3 = 87.67 \text{ MPa (tensile) } \textbf{Ans.}$$

Minimum principal (or normal) stress at point *A*,

$$\sigma_{A(min)} = \frac{\sigma_A}{2} - \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = 34.37 - 53.3 = -\,18.93 \text{ MPa}$$
$$= 18.93 \text{ MPa (compressive) } \textbf{Ans.}$$

and maximum shear stress at point *A*,

$$\tau_{A(max)} = \tfrac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$
$$= 53.3 \text{ MPa } \textbf{Ans.}$$

Stresses at point B

We know that maximum principal (or normal) stress at point *B*,

$$\sigma_{B(max)} = \frac{\sigma_B}{2} + \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\,\tau^2}\right]$$
$$= \frac{53.46}{2} + \frac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$
$$= 26.73 + 48.73 = 75.46 \text{ MPa (compressive) } \textbf{Ans.}$$

Minimum principal (or normal) stress at point *B*,

$$\sigma_{B(min)} = \frac{\sigma_B}{2} - \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\tau^2}\right]$$
$$= 26.73 - 48.73 = -\,22 \text{ MPa}$$
$$= 22 \text{ MPa (tensile) } \textbf{Ans.}$$

and maximum shear stress at point *B*,

$$\tau_{B(max)} = \tfrac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$
$$= 48.73 \text{ MPa } \textbf{Ans.}$$

UNIT 2

DESIGN FOR STATIC AND IMPACT STRENGTH

Instructional Objectives

- *Types of loading on machine elements and allowable stresses.*
- *Concept of yielding and fracture.*
- *Different theories of failure.*
- *Construction of yield surfaces for failure theories.*
- *Optimize a design comparing different failure theories*

Introduction

Machine parts fail when the stresses induced by external forces exceed their strength. The external loads cause internal stresses in the elements and the component size depends on the stresses developed. Stresses developed in a link subjected to uniaxial loading are shown in figure. Loading may be due to:

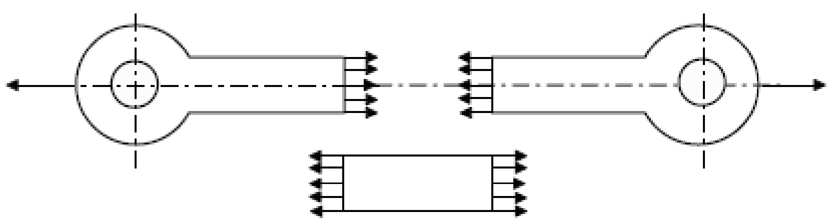
a) The energy transmitted by a machine element.

b) Dead weight.

c) Inertial forces.

d) Thermal loading.

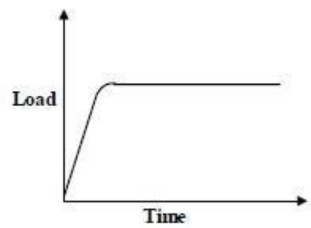
e) Frictional forces.



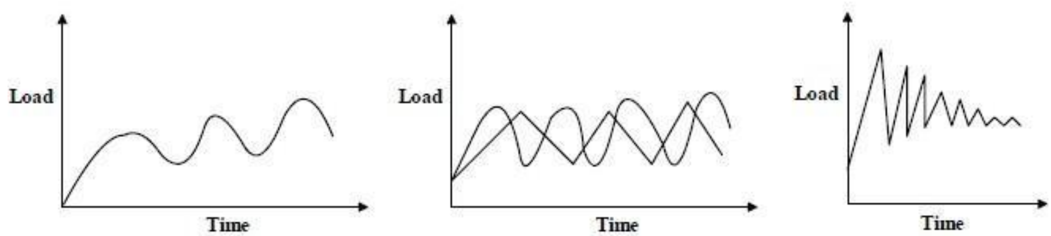
Load may be classified as:

a) Static load- Load does not change in magnitude and direction and normally increases gradually to a steady value.

b) Dynamic load- Load may change in magnitude for example, traffic of varying weight passing a bridge. Load may change in direction, for example, load on piston rod of a double acting cylinder. Vibration and shock are types of dynamic loading. **Figure** shows load v/s time characteristics for both static and dynamic loading of machine elements.



Static Loading



Dynamic Loading

Factor of Safety

Determination of stresses in structural or machine components would be meaningless unless they are compared with the material strength. If the induced stress is less than or equal to the limiting material strength then the designed component may be considered to be safe and an indication about the size of the component is obtained. The strength of various materials for engineering applications is determined in the laboratory with standard specimens. For example, for tension and compression tests a round rod of specified dimension is used in a tensile test machine where load is applied until fracture occurs. This test is usually carried out in a **Universal testing machine**. The load at which the specimen finally ruptures is known as Ultimate load and the ratio of load to original cross-sectional area is the Ultimate tress.

Similar tests are carried out for bending, shear and torsion and the results for different materials are available in handbooks. For design purpose an allowable stress is used in place of the critical stress to take into account the uncertainties including the following:

1) Uncertainty in loading.

2) In-homogeneity of materials.

3) Various material behaviors. e.g. corrosion, plastic flow, creep.

4) Residual stresses due to different manufacturing process.

5) Fluctuating load (fatigue loading): Experimental results and plot- ultimate strength depends on number of cycles.

6) Safety and reliability.

For ductile materials, the yield strength and for brittle materials the ultimate strength are taken as the critical stress. An allowable stress is set considerably lower than the ultimate strength. The ratio of ultimate to allowable load or stress is known as factor of safety i.e.

$$\mathbf{FOS} = \frac{\mathbf{Ultimate\ stress}}{\mathbf{Allowable\ stress}}$$

The ratio must always be greater than unity. It is easier to refer to the ratio of stresses since this applies to material properties.

Factor of safety = Maximum stress/ Working or design stress In case of ductile materials *e.g.* mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases, Factor of safety = Yield point stress/ Working or design stress In case of brittle materials *e.g.* cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress. Factor of safety = Ultimate stress/ Working or design stress.

Static Strength

Ideally, in designing any machine element, the engineer should have available the results of a great many strength tests of the particular material chosen. These tests should be made on specimens having the same heat treatment, surface finish, and size as the element the engineer proposes to design; and the tests should be made under exactly the same loading conditions as the part will experience in service. This means that if the part is to experience a bending load, it should be tested with a bending load. If it is to be subjected to combined bending and torsion, it should be tested under combined bending and torsion. If it is made of heat-treated

AISI 1040 steel drawn at 500◦C with a ground finish, the specimens tested should be of the same material prepared in the same manner. Such tests will provide very useful and precise information. Whenever such data are available for design purposes, the engineer can be assured of doing the best possible job of engineering.

The cost of gathering such extensive data prior to design is justified if failure of the part may endanger human life or if the part is manufactured in sufficiently large quantities. Refrigerators and other appliances, for example, have very good reliabilities because the parts are made in such large quantities that they can be thoroughly tested in advance of manufacture. The cost of making these tests is very low when it is divided by the total number of parts manufactured.

You can now appreciate the following four design categories:

1. Failure of the part would endanger human life, or the part is made in extremely large quantities; consequently, an elaborate testing program is justified during design.

2. The part is made in large enough quantities that a moderate series of tests is feasible.

3. The part is made in such small quantities that testing is not justified at all; or the design must be completed so rapidly that there is not enough time for testing.

4 The part has already been designed, manufactured, and tested and found to be unsatisfactory. Analysis is required to understand why the part is unsatisfactory and what to do to improve it.

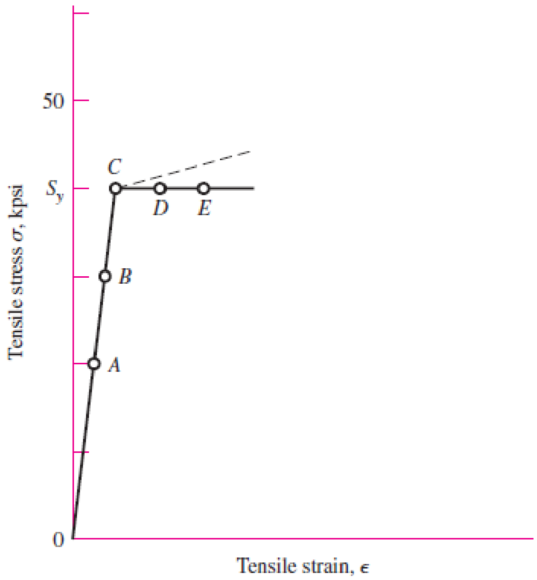
More often than not it is necessary to design using only published values of yield strength, ultimate strength, and percentage reduction in area, and percentage elongation, such as those listed in Appendix A. How can one use such meager data to design against both static and dynamic loads, two- and three-dimensional stress states, high and low temperatures, and very large and very small parts? These and similar questions will be addressed in this chapter and those to follow, but think how much better it would be tohave data available that duplicate the actual design situation.

Stress Concentration

Stress concentration is a highly localized effect. In some instances it may be due to a surface scratch. If the material is ductile and the load static, the design load may cause yielding in the critical location in the notch. This yielding can involve strain strengthening of the material and an increase in yield strength at the small critical notch location. Since the loads are static and the material is ductile, that part can carry the loads satisfactorily with no general yielding. In these cases the designer sets the geometric (theoretical) stress concentration factor K_t to unity.

The rationale can be expressed as follows. The worst-case scenario is that of an idealized non–strain-strengthening material shown in Fig. 5–6. The stress-strain curve rises linearly to the yield strength S_y , then proceeds at constant stress, which is equal to S_y . Consider a filleted rectangular bar as depicted in Fig. A–15–5, where the crosssection area of the small shank is 1 in2. If the material is ductile, with a yield point of 40 kpsi, and the theoretical stress-concentration factor (SCF) K_t is 2,

- A load of 20 kip induces a tensile stress of 20 kpsi in the shank as depicted at point *A* in Fig. 5–6. At the critical location in the fillet the stress is 40 kpsi, and the SCF is $K = \zeta_{max}/\zeta_{nom} = 40/20 = 2$.
- A load of 30 kip induces a tensile stress of 30 kpsi in the shank at point *B*. The fillet stress is still 40 kpsi (point*D*), and the SCF $K = \zeta_{max}/\zeta_{nom} = S_y/\zeta = 40/30 = 1.33$.
- At a load of 40 kip the induced tensile stress (point *C*) is 40 kpsi in the shank. At the critical location in the fillet, the stress (at point *E*) is 40 kpsi. The SCF $K = \zeta_{max}/\zeta_{nom} = S_y/\zeta = 40/40 = 1$.



For materials that strain-strengthen, the critical location in the notch has a higher Sy . The shank area is at a stress level a little below 40 kpsi, is carrying load, and is very near its failure-by-general-yielding condition. This is the reason designers do not apply Kt in *static loading* of a *ductile material* loaded elastically, instead setting Kt = 1. When using this rule for ductile materials with static loads, be careful to assure yourself that the material is not susceptible to brittle fracture (see Sec. 5–12) in the environment of use. The usual definition of geometric (theoretical) stress-concentration factor for normal stress Kt and shear stress Kts is,

$$\zeta_{max} = Kt\zeta_{nom} \qquad (a)$$
$$\eta_{max} = K_{ts}\eta_{nom} \qquad (b)$$

Since your attention is on the stress-concentration factor, and the definition of ζnom or ηnom is given in the graph caption or from a computer program, be sure the value of nominal stress is appropriate for the section carrying the load. Brittle materials do not exhibit a plastic range. A brittle material "feels" the stress concentration factor Kt or Kts , which is applied by using Eq. (*a*) or (*b*). An exception to this rule is a brittle material that inherently contains micro-discontinuity stress concentration, worse than the macro-discontinuity that the designer has in mind. Sand molding introduces sand particles, air, and water vapor bubbles. The grain structure of cast iron contains graphite flakes (with little strength), which are literally cracks introduced during the solidification process. When a tensile test on a cast iron is performed, the strength reported in the literature *includes* this stress concentration. In such cases K_t or K_{ts} need not be applied.

Failure Theories

If the failure mechanism is simple, then simple tests can give clues. Just what is simple? The tension test is uniaxial (that‟s simple) and elongations are largest in the axial direction, so strains can be measured and stresses inferred up to "failure." Just what is important: a critical stress, a critical strain, a critical energy? In the next several sections, we shall show failure theories that have helped answer some of these questions. Unfortunately, there is no universal theory of failure for the general case of material properties and stress state. Instead, over the years several hypotheses have been formulated and tested, leading to today‟s accepted practices. Being accepted, we will characterize these "practices" as *theories* as most designers do. Structural metal behavior is typically classified as being ductile or brittle, although under special situations, a material normally considered ductile can fail in a brittle manner. Ductile materials are normally classified such that $\varepsilon_f \geq 0.05$ and have an identifiable yield strength that is often the same in compression as in tension ($S_{yt} = S_{yc} = S_y$). Brittle materials, $\varepsilon_f < 0.05$, do not exhibit an identifiable yield strength, and are typically classified by ultimate tensile and compressive strengths, S_{ut} and S_{uc}, respectively (where S_{uc} is given as a positive quantity). The generally accepted theories are:

Ductile materials (yield criteria)

- Maximum shear stress (MSS)
- Distortion energy (DE)
- Ductile Coulomb-Mohr (DCM)

Brittle materials (fracture criteria)

- Maximum normal stress (MNS)
- Brittle Coulomb-Mohr (BCM)
- Modified Mohr (MM)

It would be inviting if we had one universally accepted theory for each material type, but for one reason or another, they are all used. Later, we will provide rationales for selecting a particular theory. First, we will describe the bases of these theories and apply them to some examples.

Maximum-Shear-Stress Theory for Ductile Materials

The *maximum-shear-stress theory* predicts that *yielding begins whenever the maximum shear stress in any element equals or exceeds the maximum shear stress in a tensiontest specimen of the same material when that specimen begins to yield*. The MSS theory is also referred to as the *Tresca* or *Guest theory*. Many theories are postulated on the basis of the consequences seen from tensile tests. As a strip of a ductile material is subjected to tension, slip lines (called *Lüder lines*) form at approximately 45° with the axis of the strip. These slip lines are thebeginning of yield, and when loaded to fracture, fracture lines are also seen at angles approximately 45° with the axis of tension. Since the shear stress is maximum at 45° from the axis of tension, it makes sense to think that this is the mechanism of failure. It will be shown in the next section, that there is a little more going on than this. However, it turns out the MSS theory is an acceptable but conservative predictor of failure; and since engineers are conservative by nature, it is quite often used.

Recall that for simple tensile stress, $\zeta = P/A$, and the maximum shear stress occurs on a surface 45° from the tensile surface with a magnitude of $\eta\text{max} = \zeta/2$. So the maximum shear stress at yield is $\eta\text{max} = Sy/2$. For a general state of stress, three principal stresses can be determined and ordered such that $\zeta 1 \geq \zeta 2 \geq \zeta 3$. The maximum shear stress is then $\eta\text{max} = (\zeta 1 - \zeta 3)/2$. Thus, for a general state of stress, the maximum-shear-stress theory predicts yielding when

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2} \qquad \text{or} \qquad \sigma_1 - \sigma_3 \geq S_y$$

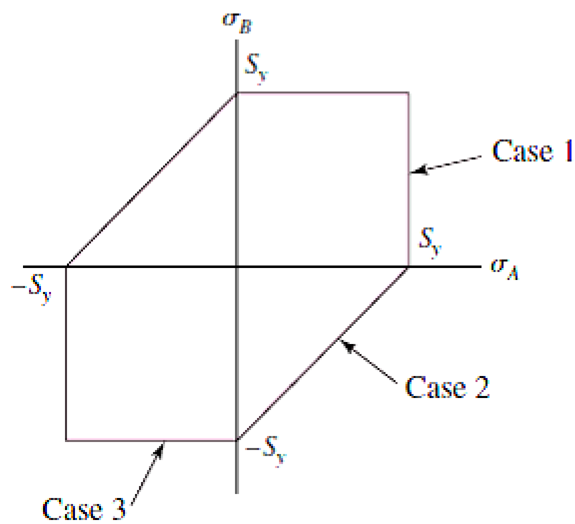
Note that this implies that the yield strength in shear is given by

$$S_{sy} = 0.5 S_y$$

Which, as we will see later is about 15 percent low (conservative). For design purposes, Above Eq. can be modified to incorporate a factor of safety, *n*. Thus,

$$\tau_{\max} = \frac{S_y}{2n} \qquad \text{or} \qquad \sigma_1 - \sigma_3 = \frac{S_y}{n}$$

Plane stress problems are very common where one of the principal stresses is zero, and the other two, ζ*A* and ζ*B*, are determined. Assuming that ζ*A* ≥ ζ*B*, there are three cases to consider for plane stress:



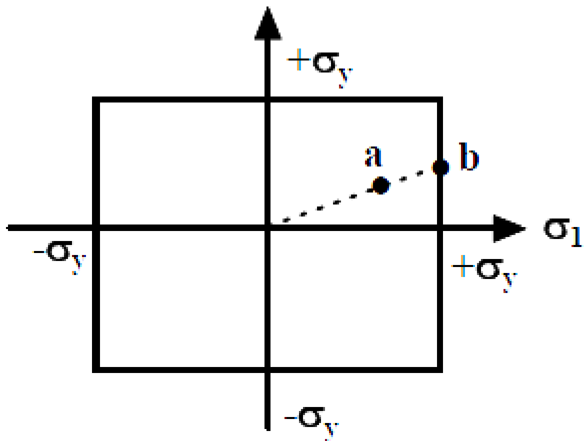
Maximum principal stress theory (Rankine theory)

According to this, if one of the principal stresses ζ1 (maximum principal stress), ζ2 (minimum principal stress) or ζ3 exceeds the yield stress, yielding would occur. In a two dimensional loading situation for a ductile material where tensile and compressive yield stress are nearly of same magnitude,

$$\sigma_1 = \pm\, \sigma_y$$

$$\sigma_2 = \pm\sigma_y$$

Yielding occurs when the state of stress is at the boundary of the rectangle.Consider, for example, the state of stress of a thin walled pressure vessel. Here ζ1= 2ζ2, ζ1 being the circumferential or hoop stress and ζ2 the axial stress. As the pressure in the vessel increases the stress follows the dotted line. At a point (say) a, the stresses are still within the elastic limit but at b, ζ1 reaches ζy although ζ2 is still less than ζy. Yielding will then begin at point b. This theory of yielding has very poor agreement with experiment. However, the theory has been used successfully for brittle materials.



Maximum principal strain theory (St. Venant‟s theory)

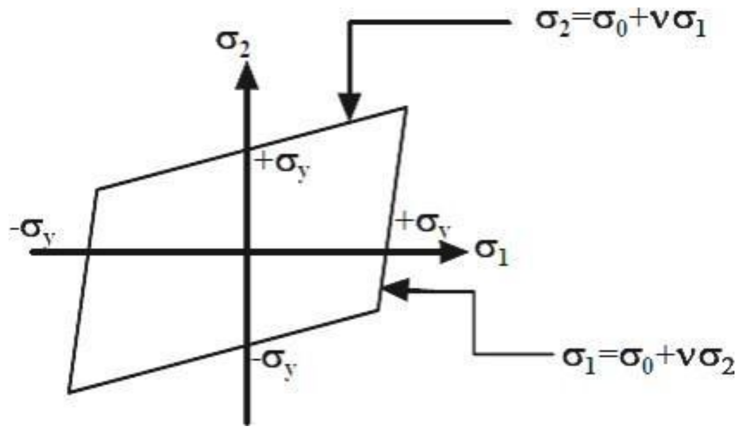
According to this theory, yielding will occur when the maximum principal strain just exceeds the strain at the tensile yield point in either simple tension or compression. If ε1 and ε2 are maximum and minimum principal strains corresponding to ζ1 and ζ2, in the limiting case,

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \qquad |\sigma_1| \geq |\sigma_2|$$

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \qquad |\sigma_2| \geq |\sigma_1|$$

This gives, $E\varepsilon_1 = \sigma_1 - \nu\sigma_2 = \pm\sigma_0$

$$E\varepsilon_2 = \sigma_2 - \nu\sigma_1 = \pm\sigma_0$$



Maximum strain energy theory (Beltrami‟s theory)

According to this theory failure would occur when the total strain energy absorbed at a point per unit volume exceeds the strain energy absorbed per unit volume at the tensile yield point. This may be given

$$\frac{1}{2}\left(\sigma_1\varepsilon_1+\sigma_2\varepsilon_2+\sigma_3\varepsilon_3\right)=\frac{1}{2}\sigma_y\varepsilon_y$$
$$\frac{1}{2}\left(\sigma_1\varepsilon_1+\sigma_2\varepsilon_2+\sigma_3\varepsilon_3\right)=\frac{1}{2}\sigma_y\varepsilon_y$$

Substituting,ε1,ε2,ε3 and εy in terms of stresses we have

$$\sigma_1{}^2+\sigma_2{}^2+\sigma_3{}^2-2\upsilon\left(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1\right)=\sigma_y{}^2$$

This may be written as

$$\left(\frac{\sigma_1}{\sigma_y}\right)^2+\left(\frac{\sigma_2}{\sigma_y}\right)^2-2\nu\left(\frac{\sigma_1\sigma_2}{\sigma_y{}^2}\right)=1$$

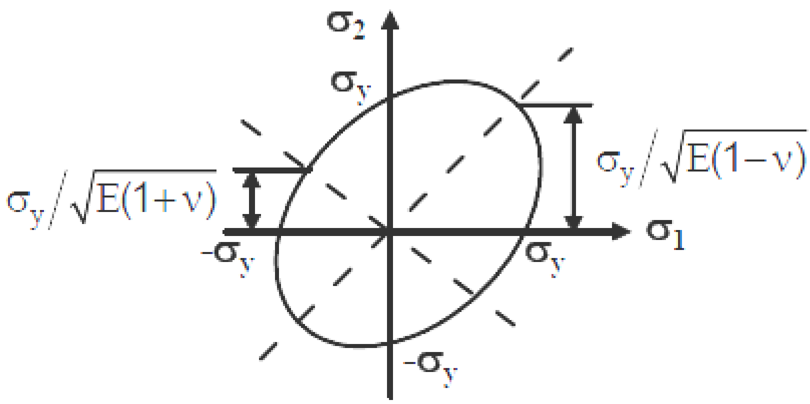


Fig: Yield surface corresponding to Maximum strain energy theory

It has been shown earlier that only distortion energy can cause yielding but in the above expression at sufficiently high hydrostatic pressure $\zeta 1 = \zeta 2 = \zeta 3 = \zeta$ (say), yielding may also occur.

Distortion energy theory(von Mises yield criterion)

According to this theory yielding would occur when total distortion energy absorbed per unit volume due to applied loads exceeds the distortion energy absorbed per unit volume at the tensile yield point. Total strain energy ET and strain energy for volume change EV can be given as

$$E_T = \frac{1}{2}\left(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3\right) \text{ and } E_V = \frac{3}{2}\sigma_{av}\varepsilon_{av}$$

Substituting strains in terms of stresses the distortion energy can be given as

$$E_d = E_T - E_V = \frac{2(1+\nu)}{6E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1\right)$$

At the tensile yield point, $\sigma_1 = \sigma_y$, $\sigma_2 = \sigma_3 = 0$ which gives

$$E_{dy} = \frac{2(1+\nu)}{6E}\sigma_y^2$$

The failure criterion is thus obtained by equating E_d and E_{dy} , which gives

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=2\sigma_y^2$$

In a 2-D situation if $\sigma_3 = 0$, the criterion reduces to

$$\sigma_1^{\ 2}+\sigma_2^{\ 2}-\sigma_1\sigma_2=\sigma_y^{\ 2}$$

i.e. $$\left(\frac{\sigma_1}{\sigma_y}\right)^2+\left(\frac{\sigma_2}{\sigma_y}\right)^2-\left(\frac{\sigma_1}{\sigma_y}\right)\left(\frac{\sigma_2}{\sigma_y}\right)=1$$

This is an equation of ellipse and the yield surface is shown in figure. This theory agrees very well with experimental results and is widely used for ductile materials.

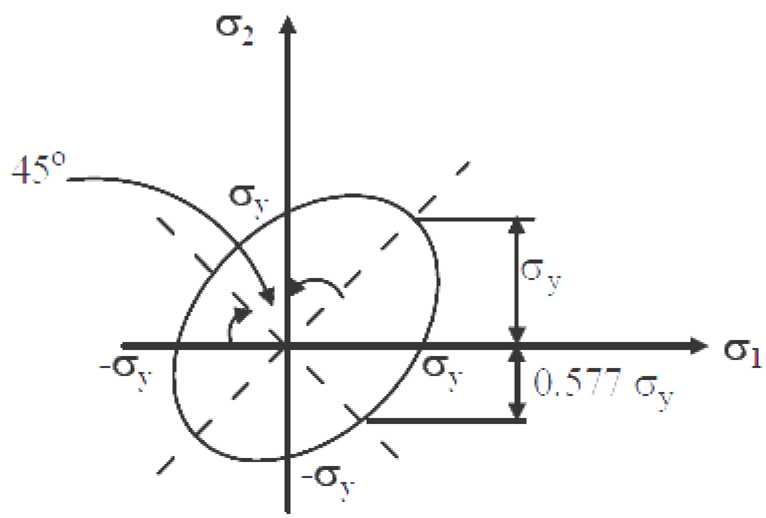


Fig: Yield surface corresponding to von Mises yield criterion.

Q.1: A shaft is loaded by a torque of 5 KN-m. The material has a yield point of 350 MPa. Find the required diameter using

(a) Maximum shear stress theory

(b) Maximum distortion energy theory

Take a factor of safety of 2.5.

Torsional shear stress induced in the shaft due to 5 KN-m torque is

$$\tau = \frac{16x(5x10^3)}{\pi d^3}$$ where d is the shaft diameter in m.

(b) Maximum shear stress theory,

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

Since $\sigma_x = \sigma_y = 0$, $\tau_{max}=25.46x10^3/d^3 = \frac{\sigma_Y}{2xF.S.} = \frac{350x10^6}{2x2.5}$

This gives d=71.3 mm.

(b) Maximum distortion energy theory

In this case $\sigma_1 = 25.46x10^3/d^3$

$\sigma_2 = -25.46x10^3/d^3$

According to this theory,

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 = 2\left(\sigma_Y / F.S\right)^2$$

Since $\sigma_3 = 0$, substituting values of σ_1, σ_2 and σ_Y

D=68 mm.

Q.2: The state of stress at a point for a material is shown in the **figure-3.1.6.1**. Find the factor of safety using (a) Maximum shear stress theory (b) Maximum distortion energy theory. Take the tensile yield strength of the material as 400 MPa.

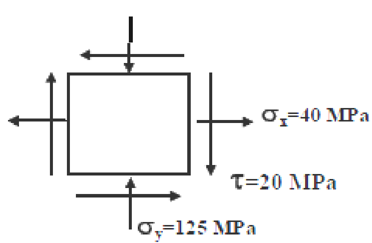


Fig: 3.1.6.1

A.2:

From the Mohr's circle, shown in **figure-3.1.6.2**

σ_1 = 42.38 MPa

σ_2 = -127.38 MPa

(a) Maximum shear stress theory

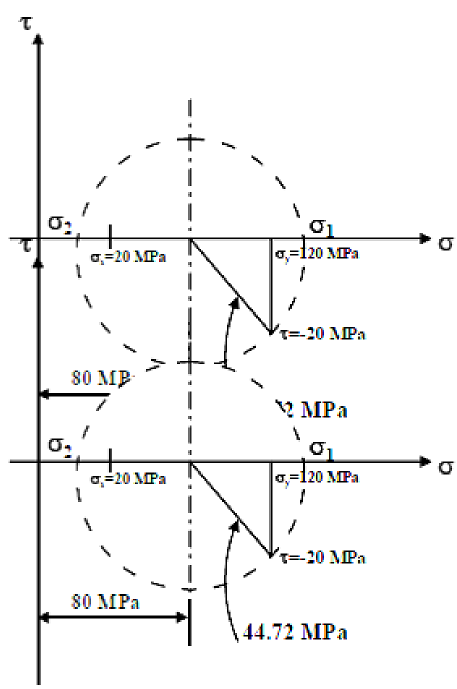
$$\frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_Y}{2\mathrm{xF.S}}$$

This gives F.S = 2.356.

(b) Maximum distortion energy theory

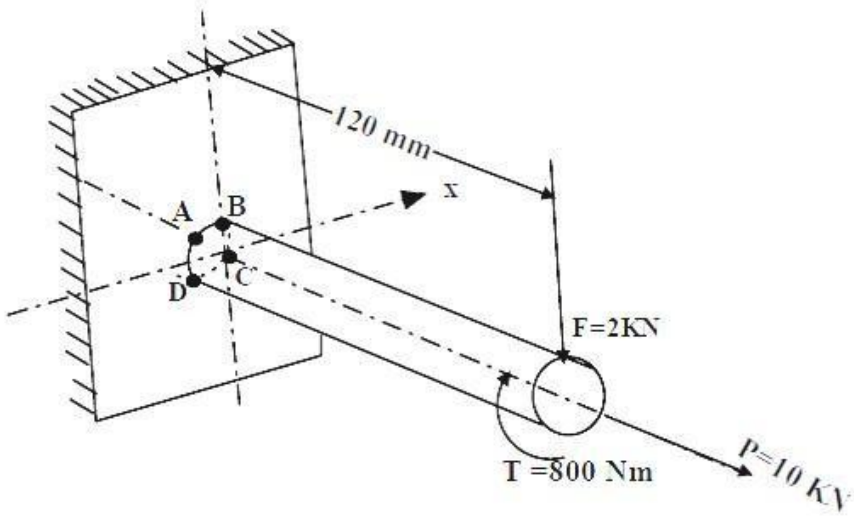
$$\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_1 - \sigma_3\right)^2 = 2\left(\sigma_Y / \mathrm{F.S}\right)^2$$

If σ_3 = 0 this gives F.S = 2.613.



3.1.6.2F

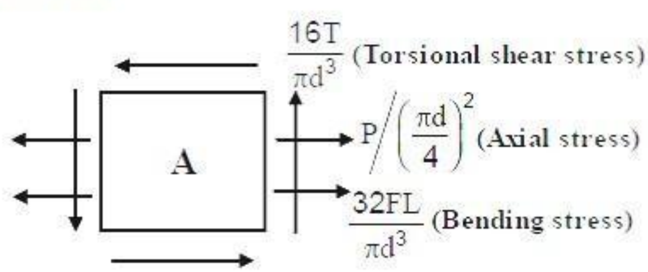
Q.3: A cantilever rod is loaded as shown in the **figure- 3.1.6.3**. If the tensile yield strength of the material is 300 MPa determine the rod diameter using (a) Maximum principal stress theory (b) Maximum shear stress theory (c) Maximum distortion energy theory.



3.1.6.3F

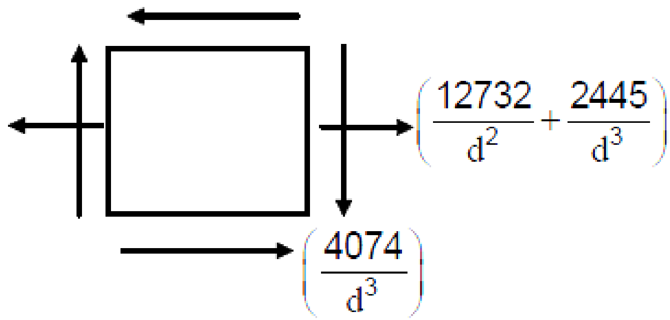
A.3:

At the outset it is necessary to identify the mostly stressed element. Torsional shear stress as well as axial normal stress is the same throughout the length of the rod but the bearing stress is largest at the welded end. Now among the four corner elements on the rod, the element A is mostly loaded as shown in **figure-3.1.6.4**



3.1.6.4F

Shear stress due to bending $\frac{VQ}{It}$ is also developed but this is neglected due to its small value compared to the other stresses. Substituting values of T, P, F and L, the elemental stresses may be shown as in **figure-3.1.6.5**:



3.1.6.5F

This gives the principal stress as

$$\sigma_{1,2}=\frac{1}{2}\left(\frac{12732}{d^2}+\frac{2445}{d^3}\right)\pm\sqrt{\frac{1}{4}\left(\frac{12732}{d^2}+\frac{2445}{d^3}\right)^2+\left(\frac{4074}{d^3}\right)^2}$$

(a) Maximum principal stress theory,

Setting $\sigma_1 = \sigma_Y$ we get d = 26.67 mm.

(b) Maximum shear stress theory,

Setting $\frac{\sigma_1-\sigma_2}{2}=\frac{\sigma_Y}{2}$, we get d = 30.63 mm.

(c) Maximum distortion energy theory,

Setting $(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2=2(\sigma_Y)^2$

We get d = 29.36 mm.

Impact Stress

Sometimes, machine members are subjected to the load with impact. The stress produced in the member due to the falling load is known as ***impact stress.*** Consider a bar carrying a load W at a height h and falling on the collar provided at the lower end, as shown in Fig.

Let A = Cross-sectional area of the bar,

E = Young's modulus of the material of the bar,

l = Length of the bar,

δl = Deformation of the bar,

P = Force at which the deflection δl is produced,

σ_i = Stress induced in the bar due to the application of impact load, and

h = Height through which the load falls.

We know that energy gained by the system in the form of strain energy

$$= \frac{1}{2} \times P \times \delta l$$

And potential energy lost by the weight

$$= W(h + \delta l)$$

Since the energy gained by the system is equal to the potential energy lost by the weight, therefore

$$\frac{1}{2} \times P \times \delta l = W\,(h + \delta l)$$

$$\frac{1}{2}\,\sigma_i \times A \times \frac{\sigma_i \times l}{E} = W\left(h + \frac{\sigma_i \times l}{E}\right) \qquad \ldots\left[\because P = \sigma_i \times A, \text{ and } \delta l = \frac{\sigma_i \times l}{E}\right]$$

$$\therefore \quad \frac{Al}{2E}(\sigma_i)^2 - \frac{Wl}{E}(\sigma_i) - Wh = 0$$

From this quadratic equation, we find that

$$\sigma_i = \frac{W}{A}\left(1 + \sqrt{1 + \frac{2\,h\,A\,E}{W\,l}}\right) \qquad \ldots \text{[Taking +ve sign for maximum value]}$$

When $h = 0$, then $\sigma_i = 2W/A$. This means that the stress in the bar when the load in applied suddenly is double of the stress induced due to gradually applied load.

Problem:

An unknown weight falls through 10 mm on a collar rigidly attached to the lower end of a vertical bar 3 m long and 600 mm2 in section. If the maximum instantaneous extension is known to be 2 mm, what is the corresponding stress and the value of unknown weight? Take $E = 200\ kN/mm^2$.

Solution. Given : $h = 10$ mm ; $l = 3$ m $= 3000$ mm ; $A = 600\ mm^2$; $\delta l = 2$ mm ; $E = 200\ kN/mm^2 = 200 \times 10^3\ N/mm^2$

Stress in the bar

Let σ = Stress in the bar.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma . l}{\delta l}$$

$$\therefore \quad \sigma = \frac{E . \delta l}{l} = \frac{200 \times 10^3 \times 2}{3000} = \frac{400}{3} = 133.3\ N/mm^2 \quad \textbf{Ans.}$$

Value of the unknown weight

Let W = Value of the unknown weight.

We know that

$$\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2h\,A\,E}{Wl}}\right]$$

$$\frac{400}{3} = \frac{W}{600}\left[1 + \sqrt{1 + \frac{2 \times 10 \times 600 \times 200 \times 10^3}{W \times 3000}}\right]$$

$$\frac{400 \times 600}{3\,W} = 1 + \sqrt{1 + \frac{800\ 000}{W}}$$

$$\frac{80\ 000}{W} - 1 = \sqrt{1 + \frac{800\ 000}{W}}$$

Squaring both sides,

$$\frac{6400 \times 10^6}{W^2} + 1 - \frac{160\,000}{W} = 1 + \frac{800\,000}{W}$$

$$\frac{6400 \times 10^2}{W} - 16 = 80 \quad \text{or} \quad \frac{6400 \times 10^2}{W} = 96$$

$$\therefore \quad W = 6400 \times 10^2 / 96 = 6666.7\ N \quad \textbf{Ans.}$$

Problem:

A wrought iron bar 50 mm in diameter and 2.5 m long transmits shock energy of 100 N-m. Find the maximum instantaneous stress and the elongation. Take E = 200 GN/m^2.

Solution. Given : $d = 50$ mm ; $l = 2.5$ m $= 2500$ mm ; $U = 100$ N-m $= 100 \times 10^3$ N-mm ; $E = 200$ GN/m^2 $= 200 \times 10^3$ N/mm^2

Maximum instantaneous stress

Let σ = Maximum instantaneous stress.

We know that volume of the bar,

$$V = \frac{\pi}{4} \times d^2 \times l = \frac{\pi}{4}(50)^2 \times 2500 = 4.9 \times 10^6 \text{ mm}^3$$

We also know that shock or strain energy stored in the body (U),

$$100 \times 10^3 = \frac{\sigma^2 \times V}{2E} = \frac{\sigma^2 \times 4.9 \times 10^6}{2 \times 200 \times 10^3} = 12.25\,\sigma^2$$

$$\therefore \quad \sigma^2 = 100 \times 10^3 / 12.25 = 8163 \quad \text{or} \quad \sigma = 90.3 \text{ N/mm}^2 \text{ Ans.}$$

Elongation produced

Let δl = Elongation produced.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\delta l / l}$$

$$\therefore \quad \delta l = \frac{\sigma \times l}{E} = \frac{90.3 \times 2500}{200 \times 10^3} = 1.13 \text{ mm} \text{ Ans.}$$

Q. A bolt is subjected to an axial pull of 10 kN and transverse shear force of 5 kN. The yield strength of the bolt material is 300 MPa. Considering the factor of safety of 2. Determine the diameter of the shaft using (i) maximum shear stress theory and (ii) distortion energy theory.

Solution: Assuming the problem for designing the bolt diameter as this problem started with the data for bolt but asked to find the diameter of the shaft.

Given: Transverse load "P_t" = 10 kN = 1000 N; Shear Load "P_s" = 5 kN =5000 N; The yield strength of the material "S_{yt}" = 300 MPa; Factor of safety "f.o.s." = 2.0;

The permissible stress: $\sigma_d = \dfrac{S_{yt}}{f.o.s} = \dfrac{300}{2.0} = 150$ MPa; -------------(1)

The axial stress: $\sigma_a = \frac{P_t}{\left(\pi d^2/4\right)} = \frac{4\times 10000}{\pi \times d^2} = \frac{12732.395}{d^2}$ MPa; ----------(2)

The transverse shear stress:

$$\tau = \frac{P_s}{\left(\pi d^2/4\right)} = \frac{4\times 5000}{\pi \times d^2} = \frac{6366.1977}{d^2} \text{ MPa; ------------(3)}$$

The principal stresses and maximum shear stress:

$$\sigma_{1,2} = \frac{\sigma_a}{2} \pm \sqrt{\left(\frac{\sigma_a}{2}\right)^2 + \tau^2} = \frac{12732.395}{2d^2} \pm \sqrt{\left(\frac{12732.395}{2d^2}\right)^2 + \left(\frac{6366.1977}{d^2}\right)^2} = \frac{(6366.1975 \pm 9003.163)}{d^2};$$

$$\sigma_1 = +\frac{15369.3605}{d^2} \text{ MPa}; \sigma_2 = \frac{-2636.9655}{d^2} \text{ MPa; -------------(4)}$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2}\left[\frac{15369.3605}{d^2} + \frac{2636.9655}{d^2}\right] = \frac{9003.163}{\mathrm{d}^2} \text{------(5)}$$

(i) **According to maximum shear stress theory:**

$$\tau_{\max} \le \frac{S_{yt}}{2\times f.o.s}$$

$$\frac{9003.163}{\mathrm{d}^2} \le \frac{300}{2\times 2}$$

$$d \ge \sqrt{\frac{4\times 9003.163}{300}} = 10.9564 \text{ mm};$$

(ii) **According to distortion energy theory:**

$$\sigma_1^2 + \sigma_2^2 - 2\sigma_1 \times \sigma_2 \le \left(\frac{S_{yt}}{f.o.s.}\right)^2;$$

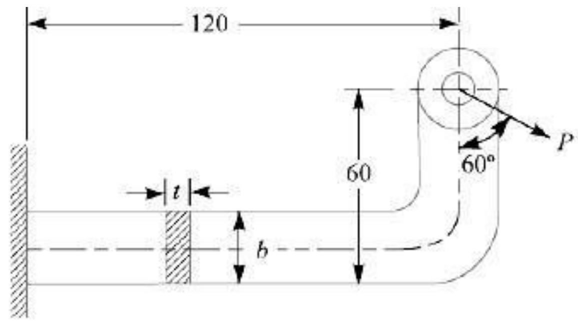
$$\left(\frac{15369.3605}{d^2}\right)^2 + \left(\frac{-2636.9655}{d^2}\right)^2 - 2\times \frac{15369.3605}{d^2} \times \frac{-2636.9655}{d^2} \le \left(\frac{300}{2.0}\right)^2$$

$$\frac{18006.326}{\mathrm{d}^2} \le 150;$$

$$d \ge \sqrt{\frac{18006.326}{150}};$$

$$d \ge 10.9564 \text{ mm};$$

Q. **A wall bracket, as shown in following figure, is subjected to a pull of $P = 5$ kN, at 60° to the vertical. The cross-section of bracket is rectangular having $b = 3t$. Determine the dimensions b and t if the stress in the material of the bracket is limited to 28 MPa.**



All dimensions in mm.

Solution: Given : $P = 6000$ N ; $\theta = 45°$; $\sigma = 60$ MPa = 60 N/mm2

Let t = Thickness of the section in mm, and

b = Depth or width of the section = 3 t

Aarea of cross-section,

$A = b \times t = 3\,t \times t = 3\,t^2$ mm^2

and section modulus,

$$Z = \frac{tb^2}{6} = \frac{3t^3}{2}$$

Horizontal component of the load,

$PH = 5000 \sin 60°$

$= 5000 \times 0.866 = 4330.13$ n

Bending moment due to horizontal component of the load,

$MH = PH \times 60 = 4330.13 \times 60 = 259807.62$ N-mm

Maximum bending stress on the upper surface due to horizontal component,

$$\sigma_{bh} = \frac{MH}{z} = \frac{259807.62 \times 2}{3t^2} = \frac{173205.81}{t^2} N/mm^2$$

Vertical component of the load,

$PV = 5000 \cos 60° = 6000 \times 0.5 = 2500$ N

Direct Shear;

$$\tau = \frac{PV}{A} = \frac{2500}{3t^2} = \frac{833.33}{t^2}\ N/mm^2$$

Bending moment due to vertical component of the load,

$MV = PV \times 60 = 2500 \times 120 = 300000$ N-mm

Maximum bending stress on the upper surface due to horizontal component,

$$\sigma_{bV} = \frac{MV}{z} = \frac{300000 \times 2}{3t^2} = \frac{200000}{t^2} N/mm^2$$

Direct tensile stress due to horizontal component

$$\sigma_d = \frac{PH}{A} = \frac{4330.13}{3t^2} = \frac{1443.38}{t^2}\ N/mm^2$$

Net normal stress

$$\sigma = \frac{173205.81}{t^2} + \frac{200000}{t^2} + \frac{1443.38}{t^2} = \frac{374649.1867}{t^2} N/mm^2$$

Now applying the maximum shear stress theory

$$\frac{1}{2}\sqrt{\left(\sigma^2 + 4\tau^2\right)} \leq 28$$

putting the values and solving the above equation for "t"

$t = 25mm$ *and* $b = 3t = 75\ mm$

<u>**Torsional Shear Stress**</u>

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to ***torsion***. The stress set up by torsion is known as ***torsional shear stress***. It is zero at the centroidal axis and maximum at the outer surface. Consider a shaft fixed at one end and subjected to a torque (*T*) at the other end as shown in Fig. As a result of this torque, every cross-section of the shaft is subjected to torsional shear stress. We have discussed above that the torsional shear stress is zero at the centroidal axis and maximum at the outer surface. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the following equation:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l} \quad \text{-------- (i)}$$

Where τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,

r = Radius of the shaft,

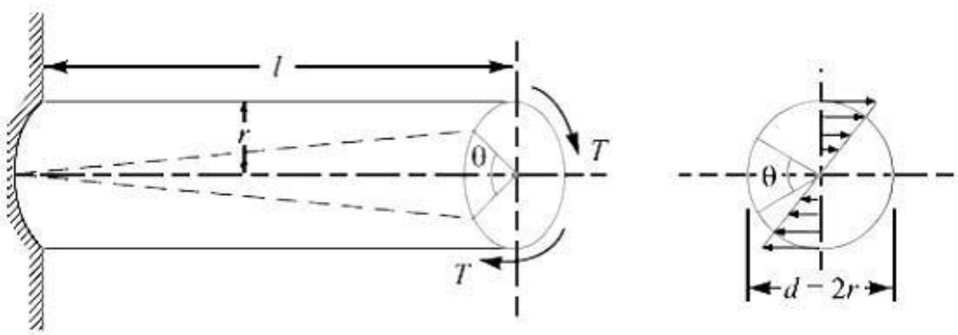
T = Torque or twisting moment,

J = Second moment of area of the section about its polar axis or polar moment of inertia,

C = Modulus of rigidity for the shaft material,

l = Length of the shaft, and

θ = Angle of twist in radians on a length l.



The above equation is known as ***torsion equation***. It is based on the following assumptions:

1. The material of the shaft is uniform throughout.

2. The twist along the length of the shaft is uniform.

3. The normal cross-sections of the shaft, which were plane and circular before twist, remain plane and circular after twist.

4. All diameters of the normal cross-section which were straight before twist, remain straight with their magnitude unchanged, after twist.

5. The maximum shear stress induced in the shaft due to the twisting moment does not exceed its elastic limit value.

Problem

A shaft is transmitting 100 kW at 160 r.p.m. Find a suitable diameter for the shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa.

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 160$ r.p.m ; $T_{max} = 1.25\ T_{mean}$; $\tau = 70$ MPa $= 70$ N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted (P),

$$100 \times 10^3 = \frac{2\,\pi\,N\,.\,T_{mean}}{60} = \frac{2\pi \times 160 \times T_{mean}}{60} = 16.76\ T_{mean}$$

$$\therefore \quad T_{mean} = 100 \times 10^3/16.76 = 5966.6 \text{ N-m}$$

and maximum torque transmitted,

$$T_{max} = 1.25 \times 5966.6 = 7458 \text{ N-m} = 7458 \times 10^3 \text{ N-mm}$$

We know that maximum torque (T_{max}),

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\ d^3$$

$$\therefore \quad d^3 = 7458 \times 10^3/13.75 = 542.4 \times 10^3 \text{ or } d = 81.5 \text{ mm \textbf{Ans.}}$$

Bending Stress

In engineering practice, the machine parts of structural members may be subjected to static or dynamic loads which cause bending stress in the sections besides other types of stresses such as tensile, compressive and shearing stresses. Consider a straight beam subjected to a bending moment M as shown in Fig.

The following assumptions are usually made while deriving the bending formula.

1. The material of the beam is perfectly homogeneous (*i.e.* of the same material throughout) and isotropic (*i.e.* of equal elastic properties in all directions).

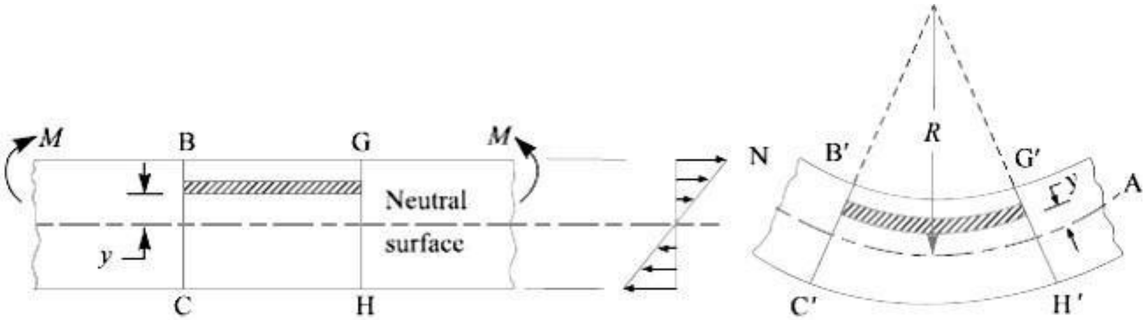
2. The material of the beam obeys Hooke's law.

3. The transverse sections (*i.e.* BC or GH) which were plane before bending remain plane after bending also.

4. Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.

5. The Young's modulus (E) is the same in tension and compression.

6. The loads are applied in the plane of bending.



A little consideration will show that when a beam is subjected to the bending moment, the fibres on the upper side of the beam will be shortened due to compression and those on the lower side will be elongated due to tension. It may be seen that somewhere between the top and bottom fibres there is a surface at which the fibres are neither shortened nor lengthened. Such a surface is called ***neutral surface***. The intersection of the neutral surface with any normal cross-section of the beam is known *as* ***neutral axis***. The stress distribution of a beam is shown in Fig. The bending equation is given by

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where M = Bending moment acting at the given section,

σ = Bending stress,

I = Moment of inertia of the cross-section about the neutral axis,

y = Distance from the neutral axis to the extreme fibre,

E = Young's modulus of the material of the beam, and

R = Radius of curvature of the beam.

From the above equation, the bending stress is given by

$$\sigma = y \times \frac{E}{R}$$

Since E and R are constant, therefore within elastic limit, the stress at any point is directly proportional to y, *i.e.* the distance of the point from the neutral axis.

Also from the above equation, the bending stress,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I/y} = \frac{M}{Z}$$

The ratio I/y is known as ***section modulus*** and is denoted by Z.

A beam of uniform rectangular cross-section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam, if depth is twice that of width.

Solution. Given: $W = 400$ N ; $L = 300$ mm ; $\sigma_b = 40$ MPa $= 40$ N/mm^2 ; $h = 2b$

The beam is shown in Fig. 5.7.

Let b = Width of the beam in mm, and

h = Depth of the beam in mm.

$\therefore$ Section modulus,

$$Z = \frac{b\,.\,h^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2\,b^3}{3}\ \text{mm}^3$$

Maximum bending moment (at the fixed end),

$$M = W.L = 400 \times 300 = 120 \times 10^3\ \text{N-mm}$$

We know that bending stress (σ_b),

$$40 = \frac{M}{Z} = \frac{120 \times 10^3 \times 3}{2\,b^3} = \frac{180 \times 10^3}{b^3}$$

$\therefore$ $b^3 = 180 \times 10^3/40 = 4.5 \times 10^3$ or $b = 16.5$ mm **Ans.**

and $h = 2b = 2 \times 16.5 = 33$ mm **Ans.**

Problem:

A cast iron pulley transmits 10 kW at 400 r.p.m. The diameter of the pulley is 1.2 metre and it has four straight arms of elliptical cross-section, in which the major axis is twice the minor axis. Determine the dimensions of the arm if the allowable bending stress is 15 MPa.

Solution. Given : $P = 10$ kW $= 10 \times 10^3$ W ; $N = 400$ r.p.m ; $D = 1.2$ m $= 1200$ mm or $R = 600$ mm ; $\sigma_b = 15$ MPa $= 15$ N/mm^2

Let T = Torque transmitted by the pulley.

We know that the power transmitted by the pulley (P),

$$10 \times 10^3 = \frac{2\,\pi\,N\,.\,T}{60} = \frac{2\pi \times 400 \times T}{60} = 42\,T$$

$$\therefore \qquad T = 10 \times 10^3/42 = 238 \text{ N-m} = 238 \times 10^3 \text{ N-mm}$$

Since the torque transmitted is the product of the tangential load and the radius of the pulley, therefore tangential load acting on the pulley

$$= \frac{T}{R} = \frac{238 \times 10^3}{600} = 396.7 \text{ N}$$

Since the pulley has four arms, therefore tangential load on each arm,

$$W = 396.7/4 = 99.2 \text{ N}$$

and maximum bending moment on the arm,

$$M = W \times R = 99.2 \times 600 = 59\,520 \text{ N-mm}$$

Let $2b$ = Minor axis in mm, and

$2a$ = Major axis in mm $= 2 \times 2b = 4b$ ...(Given)

$\therefore$ Section modulus for an elliptical cross-section,

$$Z = \frac{\pi}{4} \times a^2 b = \frac{\pi}{4}\,(2b)^2 \times b = \pi\,b^3 \text{ mm}^3$$

We know that bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{59\,520}{\pi\,b^3} = \frac{18\,943}{b^3}$$

or $b^3 = 18\,943/15 = 1263$ or $b = 10.8$ mm

$\therefore$ Minor axis, $2b = 2 \times 10.8 = 21.6$ mm **Ans.**

and major axis, $2a = 2 \times 2b = 4 \times 10.8 = 43.2$ mm **Ans.**

UNIT 3

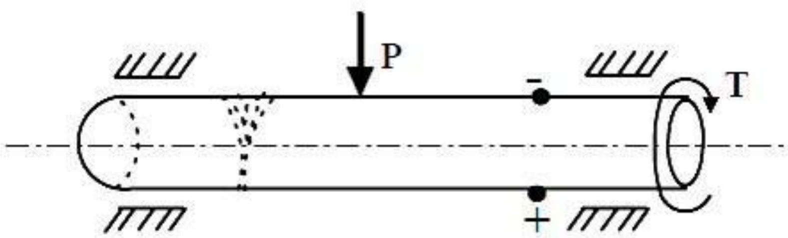
DESIGN FOR FATIGUE STRENGTH

Instructional Objectives

- *Mean and variable stresses and endurance limit.*
- *S-N plots for metals and non-metals and relation between endurance limit and ultimate tensile strength.*
- *Low cycle and high cycle fatigue with finite and infinite lives.*
- *Endurance limit modifying factors and methods of finding these factors*
- *Design of components subjected to low cycle fatigue; concept and necessary formulations.*
- *Design of components subjected to high cycle fatigue loading with finite life; concept and necessary formulations.*
- *Fatigue strength formulations; Gerber, Goodman and Soderberg equations.*

Introduction

Conditions often arise in machines and mechanisms when stresses fluctuate between a upper and a lower limit. For example in **figure-3.3.1.1**, the fiber on the surface of a rotating shaft subjected to a bending load, undergoes both tension and compression for each revolution of the shaft.



3.3.1.1F- *Stresses developed in a rotating shaft subjected to a bending load.*

Any fiber on the shaft is therefore subjected to fluctuating stresses. Machine elements subjected to fluctuating stresses usually fail at stress levels much below their ultimate strength and in many cases below the yield point of the material too. These failures occur due to very large number of stress cycle and are known as fatigue failure. These failures usually begin with a small crack which may develop at the points of discontinuity, an existing subsurface crack or surface faults. Once a crack is developed it propagates with the increase in stress cycle finally leading to failure of the component by fracture. There are mainly two characteristics of this kind of failures:

(a) Progressive development of crack.

(b) Sudden fracture without any warning since yielding is practically absent.

Fatigue failures are influenced by

(i) Nature and magnitude of the stress cycle.

(ii) Endurance limit.

(iii) Stress concentration.

(iv) Surface characteristics.

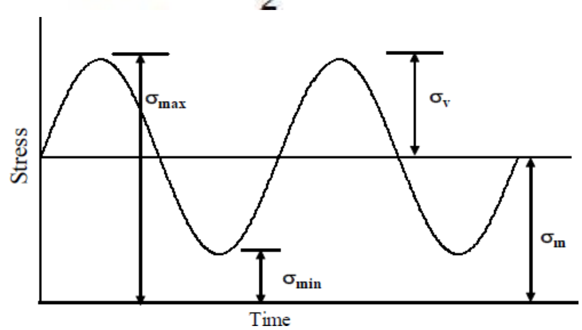
These factors are therefore interdependent. For example, by grinding and polishing, case hardening or coating a surface, the endurance limit may be improved. For machined steel endurance limit is approximately half the ultimate tensile stress. The influence of such parameters on fatigue failures will now be discussed in sequence.

Stress Cycle

A typical stress cycle is shown in **figure- 3.3.2.1** where the maximum, minimum, mean and variable stresses are indicated. The mean and variable stresses are given by

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_{variable} = \frac{\sigma_{max} - \sigma_{min}}{2}$$



3.3.2.1F- A typical stress cycle showing maximum, mean and variable stresses.

Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses; it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.

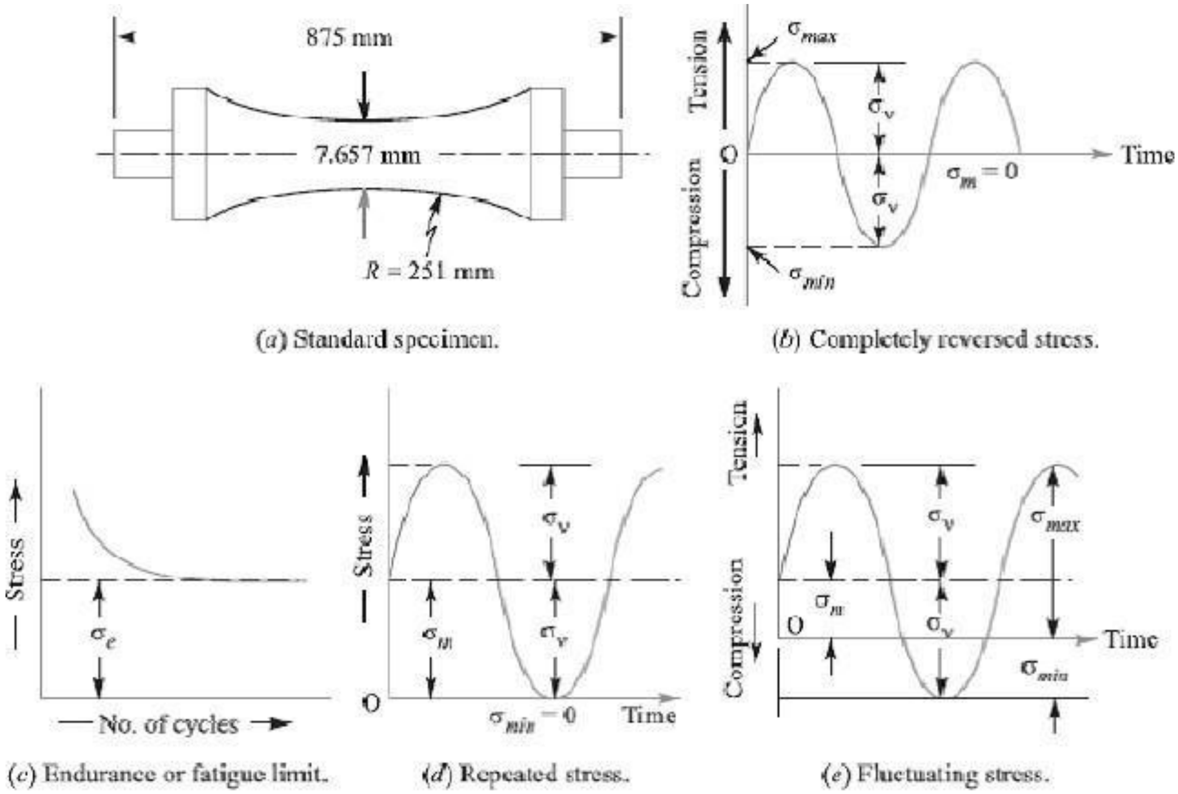


Fig.2. Time-stress diagrams.

In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig.2 (*a*), is rotated in a fatigue testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle. This is represented by a time-stress diagram as shown in Fig.2 (*b*). A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in Fig.2 (*c*). A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig.2 (*c*), the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as ***endurance*** or ***fatigue limit*** (ζ*e*). It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 107 cycles).

It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term ***endurance strength*** may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.

We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig.2 (*b*). In actual practice, many machine members undergo different range of stress than the completely reversed stress. The stress verses ***time*** diagram for fluctuating stress having values ζ_{min} and ζ_{max} is shown in Fig.2 (*e*). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component ζ_v. The following relations are derived from Fig. 2 (*e*):

1. Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

2. Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

For repeated loading, the stress varies from maximum to zero (*i.e.* $\zeta_{min} = 0$) in each cycle as shown in Fig.2 (*d*).

$$\sigma_m = \sigma_v = \frac{\sigma_{max}}{2}$$

3. Stress ratio, $R = \zeta_{max}/\zeta_{min}$. For completely reversed stresses, $R = -1$ and for repeated stresses, $R = 0$. It may be noted that R cannot be greater than unity.

4. The following relation between endurance limit and stress ratio may be used

$$\sigma'_e = \frac{3\sigma_e}{2 - R}$$

Effect of Loading on Endurance Limit—Load Factor

The endurance limit (ζe) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let, Kb = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

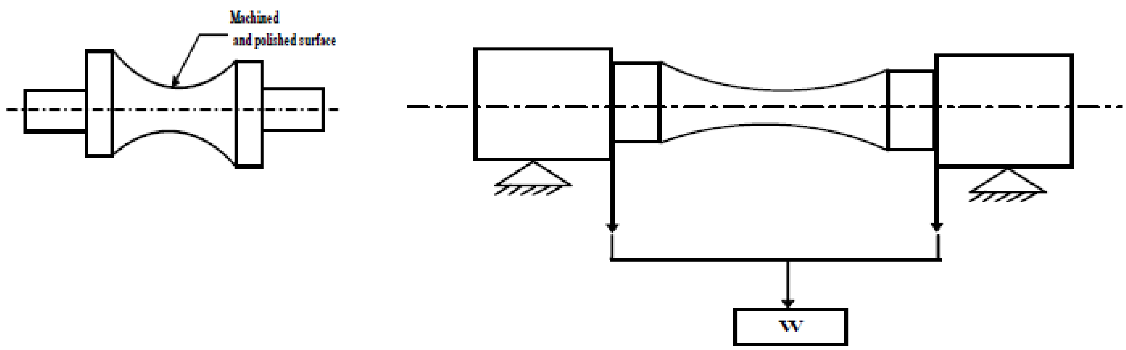
Ks = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

∴ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e K_b = \sigma_e$

Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e K_a$

and endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e K_s$

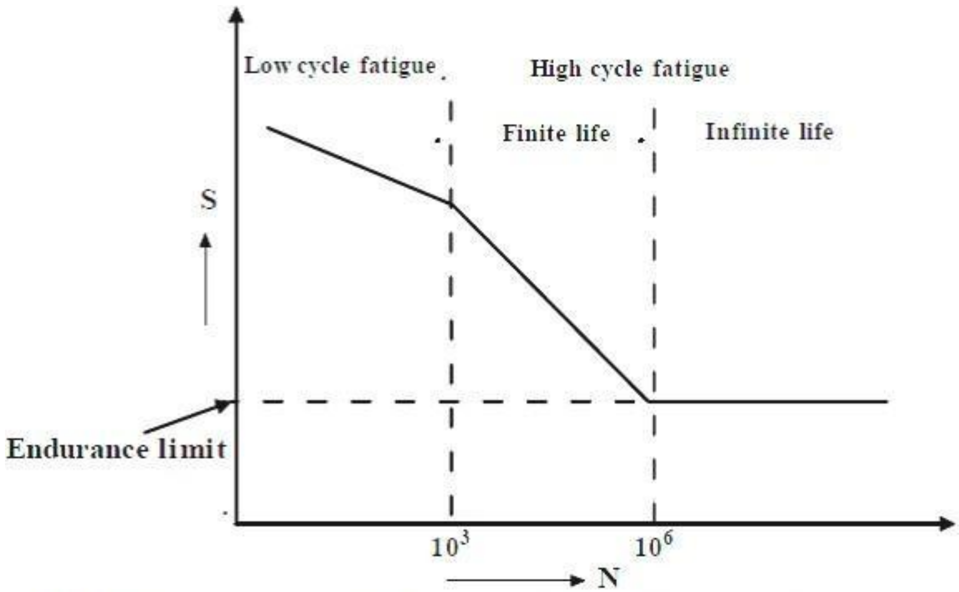
Figure- 3.3.3.1 shows the rotating beam arrangement along with the specimen.



(a) Beam specimen (b) Loading arrangement

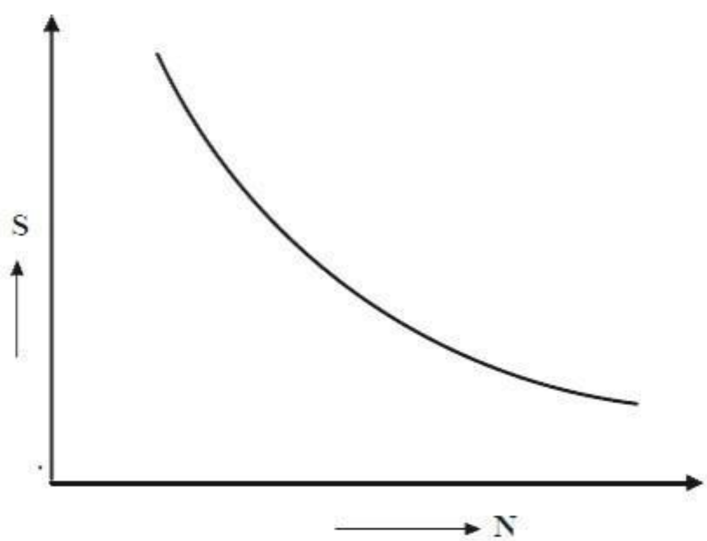
3.3.3.1F- A typical rotating beam arrangement.

The loading is such that there is a constant bending moment over the specimen length and the bending stress is greatest at the center where the section is smallest. The arrangement gives pure bending and avoids transverse shear since bending moment is constant over the length. Large number of tests with varying bending loads are carried out to find the number of cycles to fail. A typical plot of reversed stress (S) against number of cycles to fail (N) is shown in **figure-3.3.3.2**. The zone below 10^3 cycles is considered as low cycle fatigue, zone between 10^3 and 10^6 cycles is high cycle fatigue with finite life and beyond 10^6 cycles, the zone is considered to be high cycle fatigue with infinite life.



3.3.3.2F- A schematic plot of reversed stress (S) against number of cycles to fail (N) for steel.

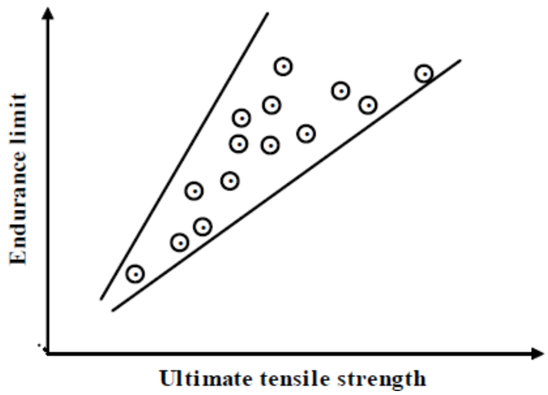
The above test is for reversed bending. Tests for reversed axial, torsional or combined stresses are also carried out. For aerospace applications and non-metals axial fatigue testing is preferred. For non-ferrous metals there is no knee in the curve as shown in **figure- 3.3.3.3** indicating that there is no specified transition from finite to infinite life.



3.3.3.3F- A schematic plot of reversed stress (S) against number of cycles to fail (N) for non-metals, showing the absence of a knee in the plot.

A schematic plot of endurance limit for different materials against the ultimate tensile strengths (UTS) is shown in **figure- 3.3.3.4**. The points lie within a narrow band and the following data is useful:

Steel	Endurance limit	~	35-60 % UTS
Cast Iron	Endurance limit	~	23-63 % UTS



F- A schematic representation of the limits of variation of endurance limit with ultimate tensile strength.

The endurance limits are obtained from standard rotating beam experiments carried out under certain specific conditions. They need be corrected using a number of factors. In general the modified endurance limit σ_e' is given by

$$\sigma_e' = \sigma_e C_1C_2C_3C_4C_5/ K_f$$

C_1 is the size factor and the values may roughly be taken as

$$C_1 = 1, \quad d \le 7.6 \text{ mm}$$
$$= 0.85, \quad 7.6 \le d \le 50 \text{ mm}$$
$$= 0.75, \quad d \ge 50 \text{ mm}$$

For large size C_1= 0.6. Then data applies mainly to cylindrical steel parts. Some authors consider 'd' to represent the section depths for non-circular parts in bending.

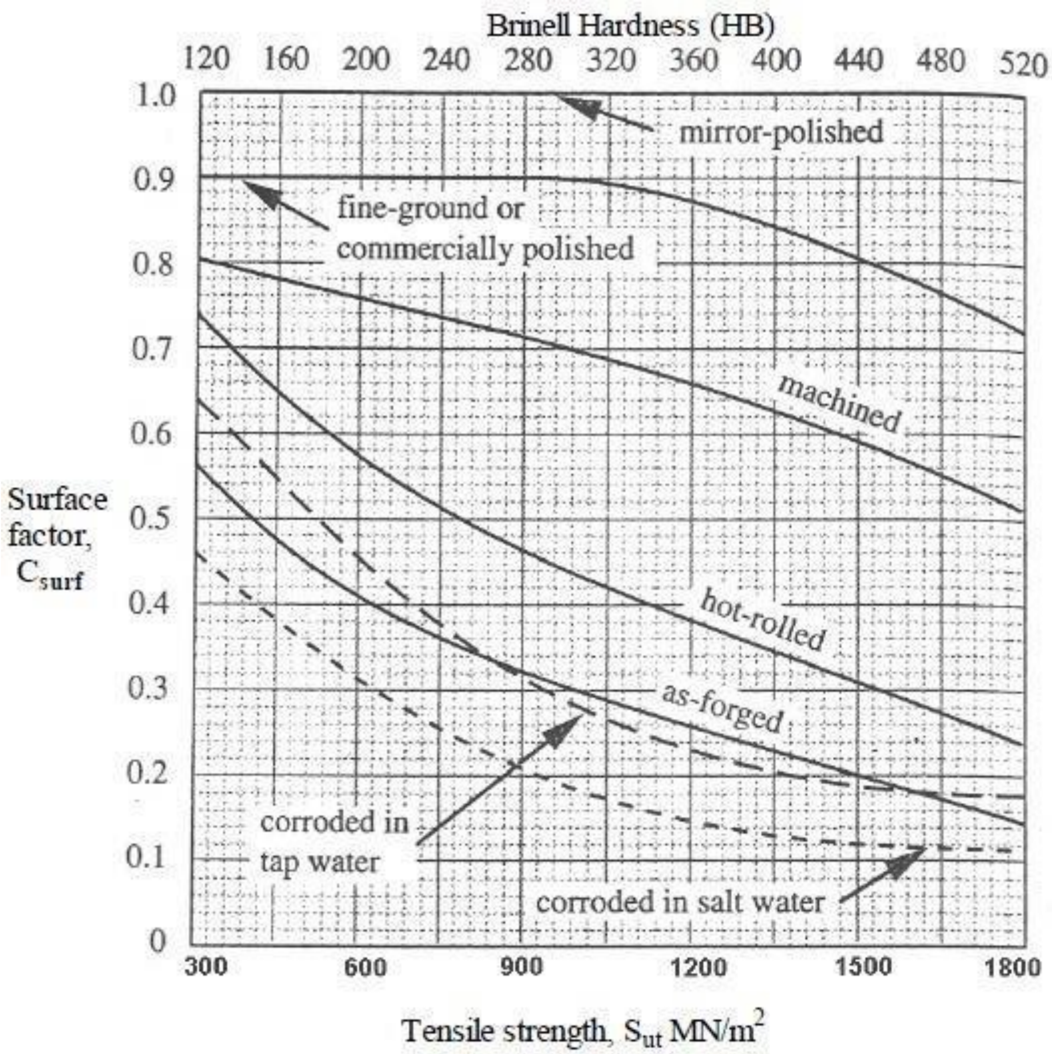
C_2 is the loading factor and the values are given as

C_2 = 1, for reversed bending load.

= 0.85, for reversed axial loading for steel parts

= 0.78, for reversed torsional loading for steel parts.

C_3 is the surface factor and since the rotating beam specimen is given a mirror polish the factor is used to suit the condition of a machine part. Since machining process rolling and forging contribute to the surface quality the plots of C_3 versus tensile strength or Brinnel hardness number for different production process, in **figure- 3.3.3.5**, is useful in selecting the value of C_3.



F- Variation of surface factor with tensile strength and Brinnel hardness for steels with different surface conditions

C_4 is the temperature factor and the values may be taken as follows:

$$C_4 = 1, \qquad \text{for } T \le 450^\circ C.$$

$$= 1-0.0058(T-450) \qquad \text{for } 450^\circ C < T \le 550^\circ C.$$

C_5 is the reliability factor

K_f is the fatigue stress concentration factor

Effect of Size on Endurance Limit—Size Factor

A little consideration will show that if the size of the standard specimen as shown in Fig.2 (*a*) is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one.

Let K_{sz} = Size factor.

Then, Endurance limit,

$$\sigma_{e2} = \sigma_{e1} \times K_{sz} \qquad \text{...(Considering surface finish factor also)}$$
$$= \sigma_{eb}.K_{sur}.K_{sz} = \sigma_e.K_b.K_{sur}.K_{sz} = \sigma_e.K_{sur}.K_{sz} \qquad (\because K_b = 1)$$
$$= \sigma_{ea}.K_{sur}.K_{sz} = \sigma_e.K_a.K_{sur}.K_{sz} \qquad \text{...(For reversed axial load)}$$
$$= \tau_e.K_{sur}.K_{sz} = \sigma_e.K_s.K_{sur}.K_{sz} \qquad \text{... (For reversed torsional or shear load)}$$

The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm. When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85. When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.

Effect of Miscellaneous Factors on Endurance Limit

In addition to the surface finish factor (K_{sur}), size factor (K_{sz}) and load factors K_b, K_a and K_s, there are many other factors such as reliability factor (K_r), temperature factor (K_t), impact factor (K_i) etc. which has effect on the endurance limit of a material. Considering all these factors, the endurance limit may be determined by using the following expressions:

1. For the reversed bending load, endurance limit,

$$\sigma'_e = \sigma_{eb}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

2. For the reversed axial load, endurance limit,

$$\sigma'_e = \sigma_{ea}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

3. For the reversed torsional or shear load, endurance limit,

$$\sigma'_e = \tau_e.K_{sur}.K_{sz}.K_r.K_t.K_i$$

In solving problems, if the value of any of the above factors is not known, it may be taken as unity.

Relation between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit (σ_e) of a material subjected to fatigue loading is a function of ultimate tensile strength (σ_u).

For steel, $\sigma_e = 0.5\ \sigma_u$;
For cast steel, $\sigma_e = 0.4\ \sigma_u$;
For cast iron, $\sigma_e = 0.35\ \sigma_u$;
For non-ferrous metals and alloys, $\sigma_e = 0.3\ \sigma_u$

Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for failure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

For steel, $\sigma_e = 0.8 \text{ to } 0.9\ \sigma_y$
σ_e = Endurance limit stress for completely reversed stress cycle, and
σ_y = Yield point stress.

Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

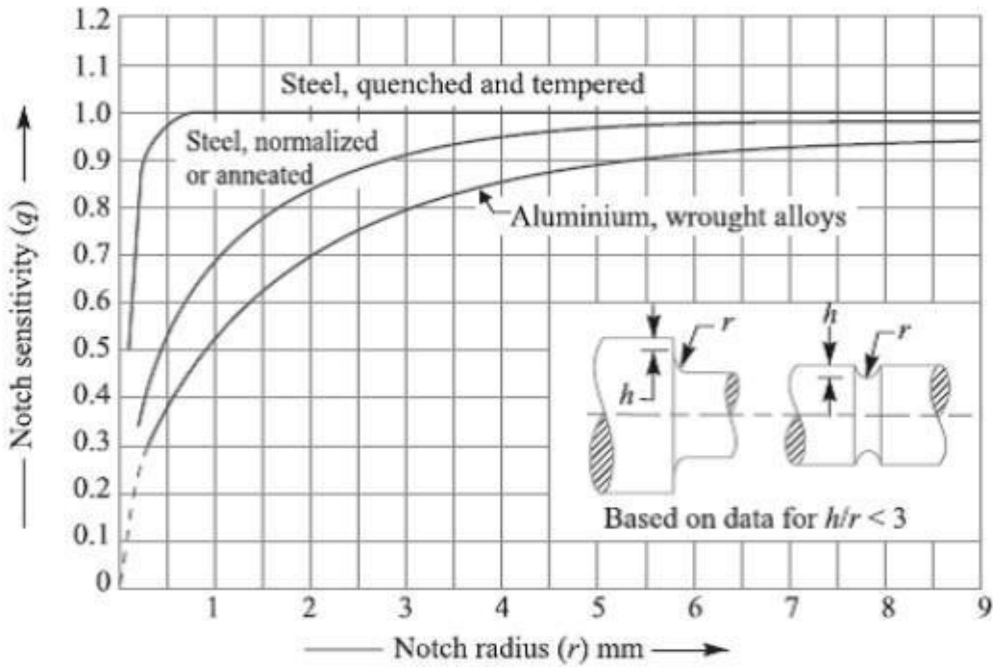
$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Notch Sensitivity

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term ***notch sensitivity*** is applied to this behaviour. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material. Since the extensive data for estimating the notch sensitivity factor (q) is not available, therefore the curves, as shown in Fig., may be used for determining the values of q for two steals. When the notch sensitivity factor q is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$$q = \frac{K_f - 1}{K_t - 1}$$

Or



$K_f = 1 + q\,(K_t - 1)$ ...[For tensile or bending stress]

And

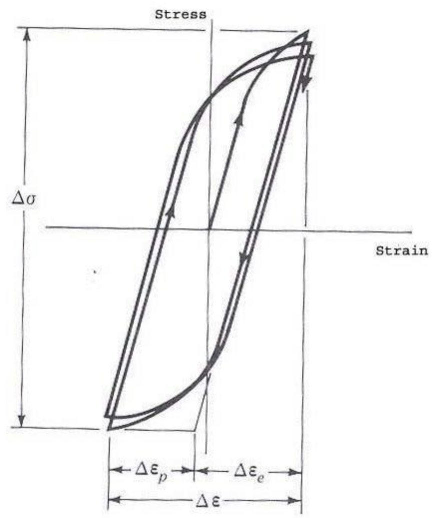
$K_{fs} = 1 + q\,(K_{ts} - 1)$ ...[For shear stress]

Where K_t = Theoretical stress concentration factor for axial or bending loading, and

K_{ts} = Theoretical stress concentration factor for torsional or shear loading.

Low Cycle Fatigue

This is mainly applicable for short-lived devices where very large overloads may occur at low cycles. Typical examples include the elements of control systems in mechanical devices. A fatigue failure mostly begins at a local discontinuity and when the stress at the discontinuity exceeds elastic limit there is plastic strain. The cyclic plastic strain is responsible for crack propagation and fracture. Experiments have been carried out with reversed loading and the true stress strain hysteresis loops are shown in **figure-3.4.1.1**. Due to cyclic strain the elastic limit increases for annealed steel and decreases for cold drawn steel. Low cycle fatigue is investigated in terms of cyclic strain. For this purpose we consider a typical plot of strain amplitude versus number of stress reversals to fail for steel as shown in **figure-3.4.1.2**.



3.4.1.1F- A typical stress-strain plot with a number of stress reversals (Ref.[4]).

Here the stress range is $\Delta\sigma$. $\Delta\varepsilon_p$ and $\Delta\varepsilon_e$ are the plastic and elastic strain ranges, the total strain range being $\Delta\varepsilon$. Considering that the total strain amplitude can be given as

$$\Delta\varepsilon = \Delta\varepsilon_p + \Delta\varepsilon_e$$

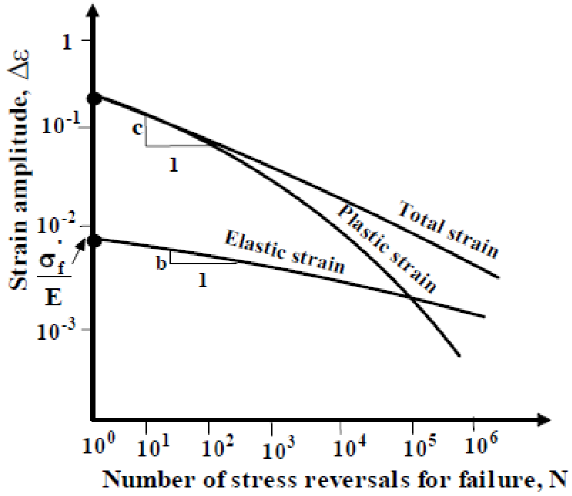
A relationship between strain and a number of stress reversals can be given as

$$\Delta\varepsilon = \frac{\sigma_f^{'}}{E}(N)^a + \varepsilon_f^{'}(N)^b$$

where σ_f and ε_f are the true stress and strain corresponding to fracture in one cycle and a, b are systems constants. The equations have been simplified as follows:

$$\Delta\varepsilon = \frac{3.5\sigma_u}{EN^{0.12}} + \left(\frac{\varepsilon_p}{N}\right)^{0.6}$$

In this form the equation can be readily used since σ_u, ε_p and E can be measured in a typical tensile test. However, in the presence of notches and cracks determination of total strain is difficult.



3.4.1.2F- Plots of strain amplitude vs number of stress reversals for failure.

High Cycle Fatigue

This applies to most commonly used machine parts and this can be analyzed by idealizing the S-N curve for, say, steel, as shown in **figure- 3.4.2.1** .

The line between 10^3 and 10^6 cycles is taken to represent high cycle fatigue with finite life and this can be given by

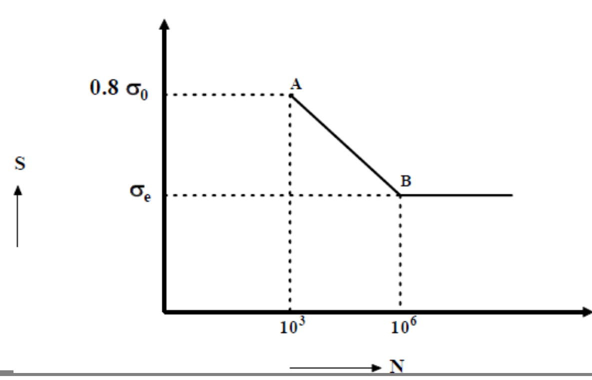
$$\log S = b \log N + c$$

where S is the reversed stress and b and c are constants.

At point A $\log(0.8\sigma_u) = b \log 10^3 + c$ where σ_u is the ultimate tensile stress

and at point B $\log \sigma_e = b \log 10^6 + c$ where σ_e is the endurance limit.

This gives $b = -\frac{1}{3}\log\frac{0.8\sigma_u}{\sigma_e}$ and $c = \log\frac{(0.8\sigma_u)^2}{\sigma_e}$



3.4.2.1F- A schematic plot of reversed stress against number of cycles to fail.

Goodman Method for Combination of Stresses:

A straight line connecting the endurance limit (σe) and the ultimate strength (σu), as shown by line *AB* in figure given below follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

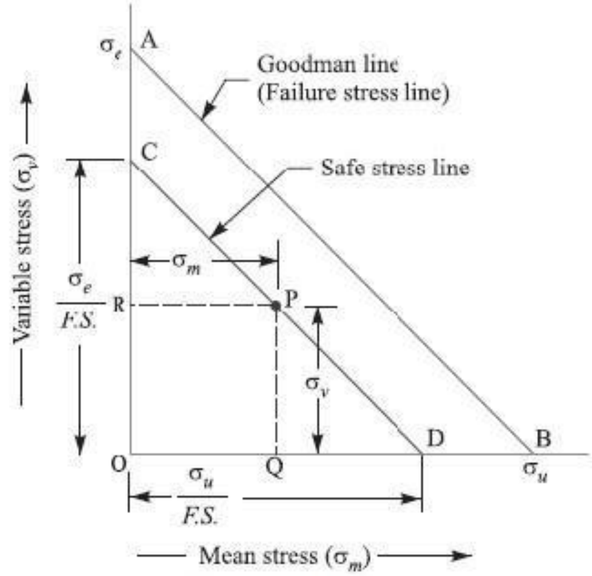


Figure 2.8

Now from similar triangles COD and PQD,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD} \qquad ...(\because QD = OD - OQ)$$

$$\therefore \qquad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_u / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u}\right]$$

$$\text{or} \qquad \frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} \qquad ...(i)$$

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads. Since many machine and structural parts that are subjected to fatigue loads contain regions of high stress concentration, therefore equation (*i*) must be altered to include this effect. In such cases, the fatigue stress concentration factor (*Kf*) is used to multiply the variable stress (σv). The equation (***i***) may now be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad ...(ii)$$

where $F.S.$ = Factor of safety,

σ_m = Mean stress,

σ_u = Ultimate stress,

σ_v = Variable stress,

σ_e = Endurance limit for reversed loading, and

K_f = Fatigue stress concentration factor.

Considering the load factor, surface finish factor and size factor, the equation (*ii*) may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_b \times K_{sur} \times K_{sz}} \qquad ...(iii)$$

$$= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \qquad ...(\because \sigma_{eb} = \sigma_e \times K_b \text{ and } K_b = 1)$$

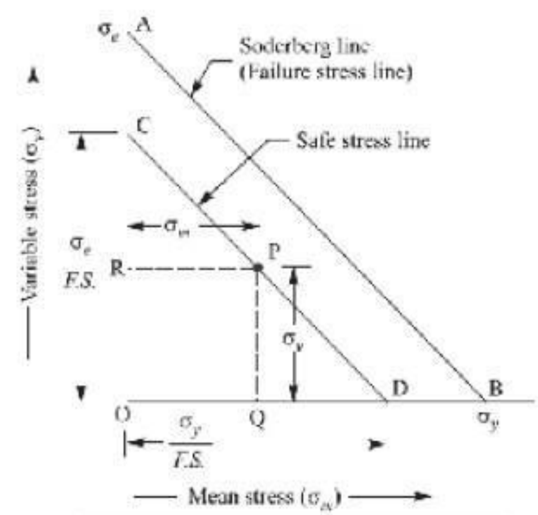
where K_b = Load factor for reversed bending load,

K_{sur} = Surface finish factor, and

K_{sz} = Size factor.

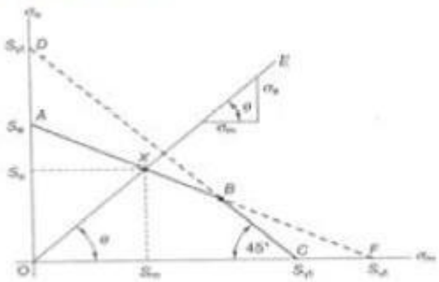
Soderberg Method for Combination of Stresses

A straight line connecting the endurance limit (σe) and the yield strength (σy), as shown by the line *AB* in following figure, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength. the line *AB* connecting σe and σy, as shown in following figure, is called ***Soderberg's failure stress line.*** If a suitable factor of safety (*F.S.*) is applied to the endurance limit and yield strength, a safe stress line *CD* may be drawn parallel to the line *AB*. Let us consider a design point *P* on the line *CD*. Now from similar triangles *COD* and *PQD*,

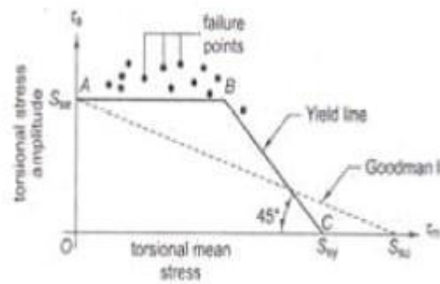


Modified Goodman Diagram:

In the design of components subjected to fluctuating stresses, the Goodman diagram is slightly modified to account for the yielding failure of the components, especially, at higher values of the mean stresses. The diagram known as modified Goodman diagram and is most widely used in the design of the components subjected to fluctuating stresses. There are two modified Goodman diagrams for the axial, normal or bending stresses and shear or torsion shear stresses separately as shown below. In the following diagrams the safe zones are ABCOA.



Modified Goodman Diagram for the Axial and Bending Stresses



Modified Goodman Diagram for the Torsional Shear Stresses

Figure 2.10

DESIGN APPROACH FOR FATIGUE LOADINGS

Design for Infinite Life

It has been noted that if a plot is made of the applied stress amplitude verses the number of reversals to failure to (S-N curve) the following behaviour is typically observed.

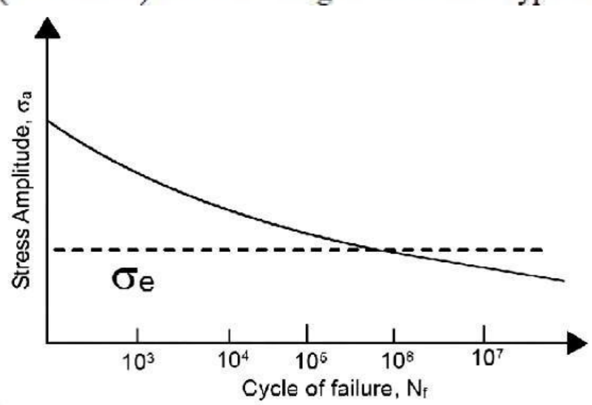


Figure 2.11

Completely Reversible Loading

If the stress is below the (the endurance limit or fatigue limit), the component has effectively infinite life. for the most steel and copper alloys. If the material does not have a well defined σ_e, Then, endurance limit is arbitrarily defined as Stress(0.35- 0.50) that gives For a known load (Moment) the section area/(modulus) will be designed such that the resulting amplitude stress will be well below the endurance limit.

Design approach can be better learnt by solving a problem.

Stress Concentration Factor

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. His irregularity in the stress distribution caused by abrupt changes of form is called ***stress concentration.*** It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc. In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in Fig. A little consideration will show that the nominal stress in the right and left hand sides will be uniform but in the region where the cross-section is changing, a re-distribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

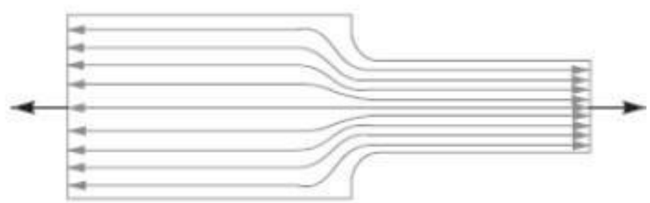


Fig. Stress concentration

Theoretical or Form Stress Concentration Factor

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area. Mathematically, theoretical or form stress concentration factor,

$$K_t = \text{Maximum stress/ Nominal stress}$$

The value of K_t depends upon the material and geometry of the part. In static loading, stress concentration in ductile materials is not so serious as in brittle materials, because in ductile materials local deformation or yielding takes place which reduces the concentration. In brittle materials, cracks may appear at these local concentrations of stress which will increase the stress over the rest of the section. It is, therefore, necessary that in designing parts of brittle materials such as castings, care should be taken. In order to avoid failure due to stress concentration, fillets at the changes of section must be provided.

In cyclic loading, stress concentration in ductile materials is always serious because the ductility of the material is not effective in relieving the concentration of stress caused by cracks, flaws, surface roughness, or any sharp discontinuity in the geometrical form of the member. If the stress at any point in a member is above the endurance limit of the material, a crack may develop under the action of repeated load and the crack will lead to failure of the member.

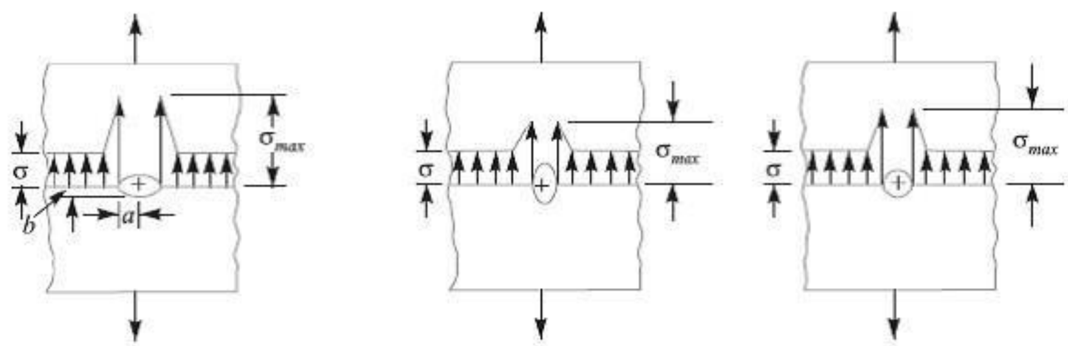
Stress Concentration due to Holes and Notches

Consider a plate with transverse elliptical hole and subjected to a tensile load as shown in Fig.1(a). We see from the stress-distribution that the stress at the point away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole. The maximum stress is given by

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right)$$

And the theoretical stress concentration factor,

$$K_t = \frac{\sigma_{max}}{\sigma} = \left(1 + \frac{2a}{r}\right)$$



$a/b = 2$
$\sigma_{max} = 5\sigma$
(*a*)

$a/b = 1/2$
$\sigma_{max} = 2\sigma$
(*b*)

$a/b = 1$
$\sigma_{max} = 3\sigma$
(*c*)

Fig.1. Stress concentration due to holes.

The stress concentration in the notched tension member, as shown in Fig. 2, is influenced by the depth a of the notch and radius r at the bottom of the notch. The maximum stress, which applies to members having notches that are small in comparison with the width of the plate, may be obtained by the following equation,

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{r}\right)$$

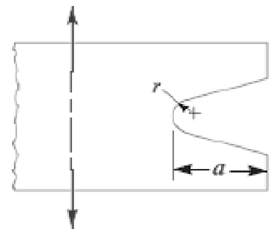


Fig.2. Stress concentration due to notches.

Methods of Reducing Stress Concentration

Whenever there is a change in cross-section, such as shoulders, holes, notches or keyways and where there is an interference fit between a hub or bearing race and a shaft, then stress concentration results. The presence of stress concentration can not be totally eliminated but it may be reduced to some extent. A device or concept that is useful in assisting a design engineer to visualize the presence of stress concentration and how it may be mitigated is that of stress flow lines, as shown in Fig.3. The mitigation of stress concentration means that the stress flow lines shall maintain their spacing as far as possible.

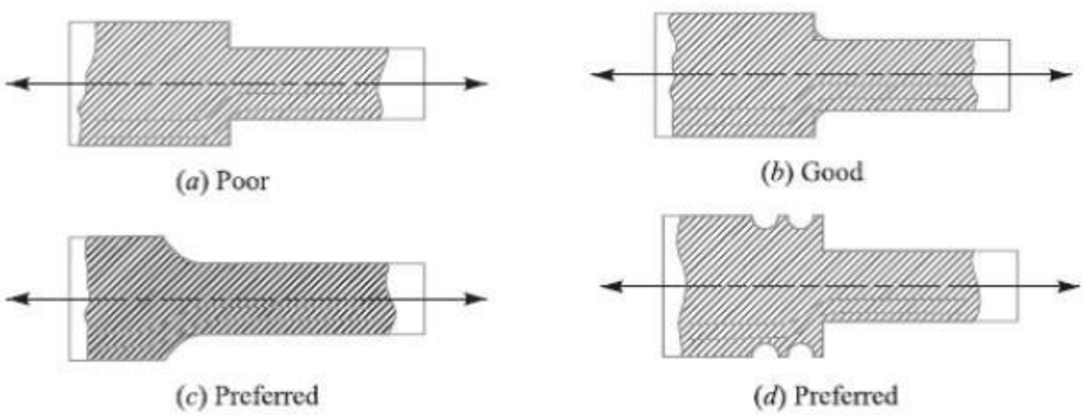


Fig.3

In Fig. 3 (a) we see that stress lines tend to bunch up and cut very close to the sharp re-entrant corner. In order to improve the situation, fillets may be provided, as shown in Fig. 3 (b) and (c) to give more equally spaced flow lines.

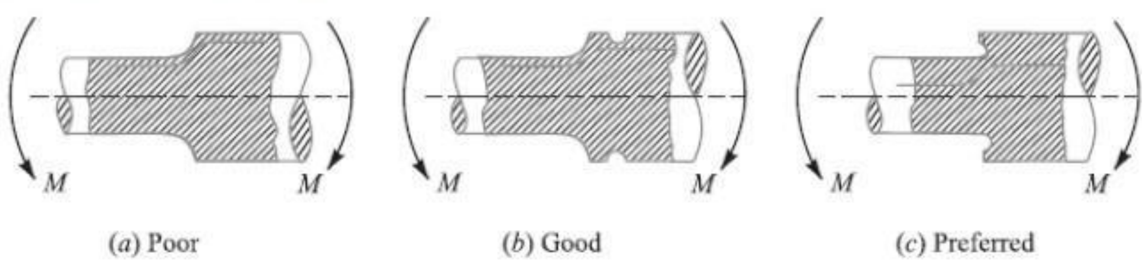


Fig. reducing stress concentration in cylindrical members with shoulders

Fig. Reducing stress concentration in cylindrical members with holes.

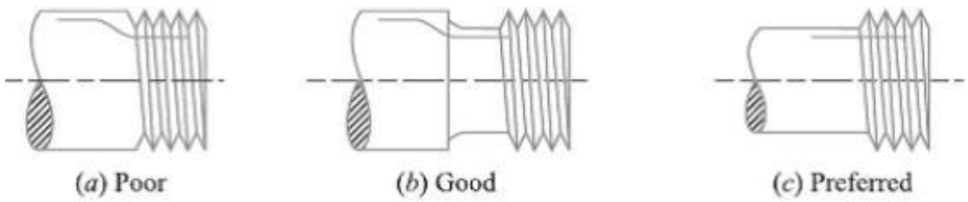


Fig. Reducing stress concentration in cylindrical members with holes

Problems:

Q. A machine component is subjected to bending stress which fluctuates between 300 N/mm² tensile and 150 N/mm² compressive in cyclic manner. Using the Goodman and Soderberg criterion, calculate the minimum required ultimate tensile strength of the material. Take the factor of safety 1.5 and the endurance limit in reversed bending as 50% of ultimate tensile strength.

Solution:

Assuming the yield strength $S_{yt} = 0.55 \times$ ultimate strength S_{ut}

Mean stress "$\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2} = \dfrac{300 + (-150)}{2} = 75\text{MPa};$

Amplitude stress "$\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \dfrac{300 - (-150)}{2} = 225\text{MPa};$

As per Goodman Relation :

$$\frac{1}{\text{f.o.s}} = \frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e}$$

As given $\quad S_e = 0.5S_{ut}$

$$\frac{1}{1.5} = \frac{75}{S_{ut}} + \frac{225}{0.5S_{ut}} \Rightarrow \frac{1}{1.5} = \frac{525}{S_{ut}} \Rightarrow S_{ut} = 787.5\text{MPa};$$

As per Soderberg Relation :

$$\frac{1}{\text{f.o.s}} = \frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e}$$

As given $\quad S_{yt} = 0.55S_{ut}$

$$\frac{1}{1.5} = \frac{75}{0.55S_{ut}} + \frac{225}{0.5S_{ut}} \Rightarrow \frac{1}{1.5} = \frac{586.36}{S_{ut}} \Rightarrow S_{ut} = 879.545\text{MPa};$$

Q. A circular bar is subjected to a completely reversed axial load of 150 kN. Determine the size of the bar for infinite life, if it is made of plain carbon steel having ultimate tensile strength of 800 N/mm^2 and yield point in tension of 600 N/mm^2. Assuming the surface finish factor as 0.80, size factor 0.85, reliability as 90%, and modifying factor for the stress concentration as 0.9.

Solution:

GIVEN:

Maximum Axial Load "P_{max}" = +150kN; Minimum Axial Load "P_{min}" = −150kN;

Ultimate tensile strength of the material of the bar "S_{ut}" = 800 N/mm^2;

Yield point in tension of the material of the bar "S_{yt}" = 600 N/mm^2;

Surface finish factor "k_a" = 0.80; Size factor "k_b" = 0.85; Reliability factor "k_c" = 0.90; Modifying the stress concentration factor "k_e" = 0.90.

ASSUMING:

The temperature factor $k_d = 1.0$; & miscelleneous factor $k_g = 1.0$;

Factor of safety = 1.0;

Endurance Limit of the material:

$S_e' = 0.5 \times S_{ut} = 0.5 \times 800 = 400\,\text{MPa}$

Modified Endurance Limit of the Material of the Bar:

$S_e = k_a \times k_b \times k_c \times k_d \times k_e \times k_g \times S_e' = 0.80 \times 0.85 \times 0.90 \times 1.0 \times 0.9 \times 1.0 \times 400 = 220.32\,\text{MPa};$

Amplitude and Mean Normal Stresses:

$$\text{Amplitude Load "}P_a\text{"} = \frac{P_{max} - P_{min}}{2} = \frac{150-(-150)}{2} = 150\,\text{kN};$$

$$\text{Mean Load "}P_m\text{"} = \frac{P_{max} + P_{min}}{2} = \frac{150+(-150)}{2} = 0\,\text{kN};$$

$$\text{Amplitude Stress "}\sigma_a\text{"} = \frac{4 \times P_a}{\pi \times d^2} = \frac{4 \times 150}{\pi \times d^2} = \frac{190.9859}{d^2}\,\text{kN/mm}^2 = \frac{190985.9}{d^2}\,\text{N/mm}^2;$$

Mean Stress "σ_m" = 0 N/mm^2;

Using Modified Goodman Diagram:

The Load Line becomes the amplitude axis.

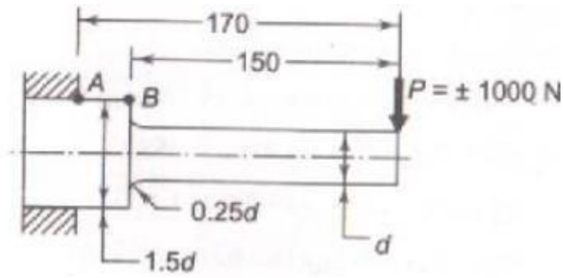
Hence the design equation may be written for infinite life as:

$$S_e \geq \sigma_a \Rightarrow 220.32 = \frac{190985.9}{d^2};$$

$$d \geq \sqrt{\frac{190985.9}{220.32}} \Rightarrow d \geq 29.44$$

$d = 30\text{mm}.$

(b) **A cantilever beam made of cold drawn steel 20C8 (S_{ut} =540 N/mm^2) is subjected to a completely reversed load of 1000 N as shown in below figure. The corrected endurance limit for the material of the beam may be taken as 123.8 N/mm^2. Determine the diameter "d" of the beam for a life of 10000 cycles.**



Solution:

GIVEN:

Maximum Axial Load "P_{max}" = +1000 N;

Minimum Axial Load "P_{min}" = −1000 N;

Ultimate tensile strength of the material of the bar "S_{ut}" = 540 N/mm^2;

Corrected Endurance Limit "S_e" = 123.8 N / mm^2

USING THE S-N DIAGRAM:

The values of various points

$0.9S_{ut} = 0.9 \times 540 = 486\ \text{N/mm}^2;$

$\log_{10}(0.9S_{ut}) = \log_{10}(486) = 2.6866;$

$\log_{10}(S_e) = \log_{10}(123.8) = 2.0927;$

$\log_{10}(N) = \log_{10}(10000) = 4.0$

From the S-N Diagram:

$$\overline{AE} = \frac{\overline{AD} \times \overline{EF}}{\overline{DB}} = \frac{(2.6866 - 2.0927) \times (4-3)}{(6-3)} = 0.198$$

Therefor

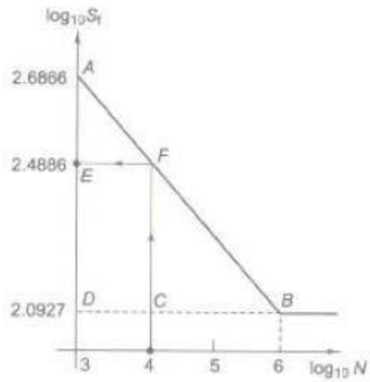
$\log_{10} S_f = 2.6866 - \overline{AE} = 2.6866 - 0.198 = 2.4886;$

$S_f = 308.03\ \text{N/mm}^2;$

And

$$S_f = \sigma_b = \frac{32M_b}{\pi d^3} \Rightarrow d^3 = \frac{32 \times M_b}{\pi \times S_f} = \frac{32 \times (1000 \times 150)}{\pi \times 308.3}$$

$d = 17.05$ mm.



$\log_{10} S_f$
2.6866
A
2.4886
E
F
2.0927
D
C
B
3
4
5
6
$\log_{10} N$

Q. **A flat bar as shown in the figure 1 is subjected to an axial load F equal to 500 N. Assuming that the stress in the bar is limited to 200 MPa, determine the thickness of the bar. All dimensions are in mm.**

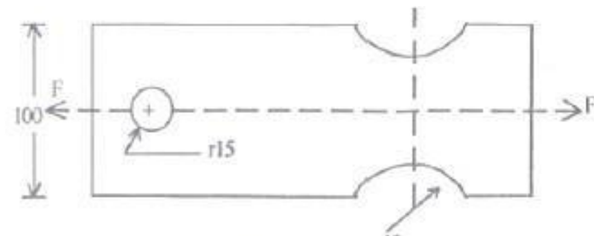


Figure 1

Solution: The stress concentration factor at circular hole k_t =2.35

The stress concentration factor at circular hole k_t =1.78

Hence the critical section is at the section of circular hole. The stress magnitude induced at this location

$$\sigma = \frac{F}{t \times (100-30)} = \frac{500}{t \times 70} = \frac{7.143}{t} N/mm^2;$$

For successful design

$$\frac{7.143}{t} \leq 200;$$
$$t \geq 0.036 mm;$$

However the minimum cross section area "A" =62t mm^2;

$$\sigma = \frac{F}{62t} = \frac{500}{t \times 62} = \frac{8.06452}{t} N/mm^2;$$

For successful design

$$\frac{8.06452}{t} \leq 200;$$
$$t \geq 0.0403 mm;$$

Hence the thickness of the plate = 0.0403 mm.

Q. **A forged steel bar 50 mm in diameter is subjected to a reversed bending stress of 300 MPa. The bar is made of 40C8. Calculate the life of the bar for a reliability of 90%**

Given: ***The material 40C8,***

The diameter of the shaft = d=50mm;

Reversed bending stress = 300MPa.

Reliability = 90%

Assuming: *σ_{ut}= 600MPa ; σ_{yt}=380 MPa Fatigue stress concentration factor=1.612*

Assuming the size factor $Ksz = 0.85$;

Surface finish factor $Ksur = 0.89$

Reliability factor K_{re}=0.892

Endurance Limit:

$$S'_e = 0.5\sigma_{ut} \qquad \because \sigma_{ut} < 1400MPa;$$
$$S'_e = 0.5 \times 600 = 300MPa;$$

Modified Endurance Limit:

$$S_e = K_{sz} \times K_{sur} \times K_{re} \times \frac{1}{K_f} \times S'_e$$
$$= 0.85 \times 0.89 \times 0.892 \times \frac{1}{1.612} \times 300 = 125.583 \text{ MPa}$$

Using the S-N diagram

$$\log_{10}(S_{ut}) = \log_{10}(600) = 2.778;$$
$$0.9 \times \log_{10}(S_{ut}) = 2.500$$
$$\log_{10}(S_e) = \log_{10}(125.583) = 2.099;$$
$$\log_{10}(\sigma_a) = \log_{10}(300) = 2.477;$$
$$\frac{2.50 - 2.099}{6-3} = \frac{2.477 - 2.099}{6 - \log_{10} N}$$
$$6 - \log_{10} N = 2.82793$$
$$\log_{10} N = 6 - 2.82793 = 3.17207$$
$$N = 1486.1752 \text{ reversals}$$

Q. A shaft subjected to bending moment varying from -200 N m to +500 N m and a varying torque from 50 N m to 175 N m. If material of the shaft is 30C8, stress concentration factor is 1.85, notch sensitivity is 0.95 reliability 99.9% and factor of safety is 1.5, find the diameter of the shaft.

Solution: Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{500 + (-200)}{2} = 150N - m\ ;$$

Amplitude or variable bending moment

$$M_a = \frac{M_{max} - M_{min}}{2} = \frac{500 - (-200)}{2} == 350N - m$$

Mean or average torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{175 + (50)}{2} = 112.5N - m\ ;$$

Amplitude or variable torque

$$T_a = \frac{T_{max} - T_{min}}{2} = \frac{175 - (50)}{2} == 62.5N - m$$

Equivalent mean and amplitude bending moments

$$M_{em} = \frac{1}{2}\left[M_m + \sqrt{M_m^2 + T_m^2}\right] = \frac{1}{2}\left[150 + \sqrt{150^2 + 112.5^2}\right] = 168.75\mathrm{N\text{-}m} = 168750\ \mathrm{N\text{-}mm};$$

$$M_{am} = \frac{1}{2}\left[M_a + \sqrt{M_a^2 + T_a^2}\right] = \frac{1}{2}\left[350 + \sqrt{350^2 + 62.5^2}\right] = 352.77\mathrm{N\text{-}m} = 352770\ \mathrm{N\text{-}mm};$$

Mean or average bending stress,

$$\sigma_m = \frac{32M_{em}}{\pi \times d^3} = \frac{32 \times 168750}{\pi \times d^3} = \frac{1718874}{d^3};\ ;$$

Amplitude or variable bending moment

$$\sigma_a = \frac{32M_a}{\pi \times d^3} = \frac{32 \times 352.77}{\pi \times d^3} = \frac{3593286}{d^3};$$

Material properties: σ_{ut}= 490MPa ; σ_{yt}=270 MPa (assumed)

Given Notch sensitivity $q = 0.95$;

Assuming the size factor $Ksz = 0.85$;

Surface finish factor $Ksur = 0.89$

Reliability factor K_{re}=0.75 corresponding to 99.9% reliability (Assumed)

The fatigue stress concentration factor

$$K_f = 1 + q(K_t - 1) = 1 + 0.95(1.85 - 1) = 1.8075$$

Endurance Limit:

$$S'_e = 0.5\sigma_{ut} \qquad \because \sigma_{ut} < 1400MPa;$$

$$S'_e = 0.5 \times 490 = 245MPa;$$

Modified Endurance Limit:

$$S_e = K_{sz} \times K_{sur} \times K_{re} \times \frac{1}{K_f} \times S'_e = 0.85 \times 0.89 \times 0.75 \times \frac{1}{1.8075} \times 245 = 76.91\text{MPa}$$

We know that according to Soderberg formula;

$$\frac{1}{f.o.s.} = \frac{\sigma_m}{\sigma_{yt}} + \frac{\sigma_a}{S_e};$$

$$\frac{1}{1.5} = \frac{\frac{1718874}{d^3}}{270} + \frac{\frac{3593286}{d^3}}{76.91};$$

$$\frac{d^3}{1.5} = \frac{1718874}{270} + \frac{3593286}{76.91};$$

$$d^3 = 1.5 \times [100519 + 6083178] = 79630.291$$

$$\text{d} = 43.022 \text{ mm} \approx 45 \text{ mm};$$

We know that according to Goodman's formula

$$\frac{1}{f.o.s.} = \frac{\sigma_m}{\sigma_{ut}} + \frac{\sigma_a}{S_e};$$

$$\frac{1}{1.5} = \frac{\frac{1718874}{d^3}}{490} + \frac{\frac{3593286}{d^3}}{76.91};$$

$$\frac{d^3}{1.5} = \frac{1718874}{490} + \frac{3593286}{76.91};$$

$$d^3 = 75342.85$$

$$\text{d} = 42.235\text{mm} \approx 45 \text{ mm};$$

Hence the shaft diameter is 45 mm. Ans.

Problem: Determine the thickness of a 120 mm wide uniform plate for safe continuous operation if the plate is to be subjected to a tensile load that has a maximum value of 250 kN and a minimum value of 100 kN. The properties of the plate material are as follows: Endurance limit stress = 225 MPa, and Yield point stress = 300 MPa. The factor of safety based on yield point may be taken as 1.5.

Let t = Thickness of the plate in mm.

$\therefore$ Area, $A = b \times t = 120\, t \text{ mm}^2$

We know that mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{250 + 100}{2} = 175 \text{ kN} = 175 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{175 \times 10^3}{120t} \text{ N/mm}^2$$

$$\text{Variable load, } W_v = \frac{W_{max} - W_{min}}{2} = \frac{250 - 100}{2} = 75 \text{ kN} = 75 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Variable stress, } \sigma_v = \frac{W_v}{A} = \frac{75 \times 10^3}{120t} \text{ N/mm}^2$$

According to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{1.5} = \frac{175 \times 10^3}{120t \times 300} + \frac{75 \times 10^3}{120t \times 225} = \frac{4.86}{t} + \frac{2.78}{t} = \frac{7.64}{t}$$

$\therefore$ $t = 7.64 \times 1.5 = 11.46$ say 11.5 mm **Ans.**

Problem:

Determine the diameter of a circular rod made of ductile material with a fatigue strength (complete stress reversal), σ_e = 265 MPa and a tensile yield strength of 350 MPa. The member is subjected to a varying axial load from $W_{min} = -300 \times 10^3$ N to $W_{max} = 700 \times 10^3$ N and has a stress concentration factor = 1.8. Use factor of safety as 2.0.

Let d = Diameter of the circular rod in mm.

$$\therefore \quad \text{Area, } A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that the mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{700 \times 10^3 + (-300 \times 10^3)}{2} = 200 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{200 \times 10^3}{0.7854\, d^2} = \frac{254.6 \times 10^3}{d^2} \text{ N/mm}^2$$

Variable load, $W_v = \frac{W_{max} - W_{min}}{2} = \frac{700 \times 10^3 - (-300 \times 10^3)}{2} = 500 \times 10^3$ N

$\therefore$ Variable stress, $\sigma_v = \frac{W_v}{A} = \frac{500 \times 10^3}{0.7854\, d^2} = \frac{636.5 \times 10^3}{d^2}$ N/mm^2

We know that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e}$$

$$\frac{1}{2} = \frac{254.6 \times 10^3}{d^2 \times 350} + \frac{636.5 \times 10^3 \times 1.8}{d^2 \times 265} = \frac{727}{d^2} + \frac{4323}{d^2} = \frac{5050}{d^2}$$

$\therefore$ $d^2 = 5050 \times 2 = 10\,100$ or $d = 100.5$ mm **Ans.**

Problem:

A circular bar of 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN and a maximum value of 50 kN. Determine the diameter of bar by taking a factor of safety of 1.5, size effect of 0.85, surface finish factor of 0.9. The material properties of bar are given by: ultimate strength of 650 MPa, yield strength of 500 MPa and endurance strength of 350 MPa.

Solution. Given : l = 500 mm ; W_{min} = 20 kN = 20 × 10^3 N ; W_{max} = 50 kN = 50 × 10^3 N ; $F.S.$ = 1.5 ; K_{sz} = 0.85 ; K_{sur} = 0.9 ; σ_u = 650 MPa = 650 N/mm^2 ; σ_y = 500 MPa = 500 N/mm^2 ; σ_e = 350 MPa = 350 N/mm^2

Let d = Diameter of the bar in mm.

We know that the maximum bending moment,

$$M_{max} = \frac{W_{max} \times l}{4} = \frac{50 \times 10^3 \times 500}{4} = 6250 \times 10^3 \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times l}{4} = \frac{20 \times 10^3 \times 500}{4} = 2550 \times 10^3 \text{ N-mm}$$

$\therefore$ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{6250 \times 10^3 + 2500 \times 10^3}{2} = 4375 \times 10^3 \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{6250 \times 10^3 - 2500 \times 10^3}{2} = 1875 \times 10^3 \text{ N-mm}$$

Section modulus of the bar,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

$\therefore$ Mean or average bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{4375 \times 10^3}{0.0982\, d^3} = \frac{44.5 \times 10^6}{d^3} \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{1875 \times 10^3}{0.0982 d^3} = \frac{19.1 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 650} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \quad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{68 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{139 \times 10^3}{d^3}$$

$\therefore$ $d^3 = 139 \times 10^3 \times 1.5 = 209 \times 10^3$ or $d = 59.3$ mm

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 500} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \quad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{89 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{160 \times 10^3}{d^3}$$

$\therefore$ $d^3 = 160 \times 10^3 \times 1.5 = 240 \times 10^3$ or $d = 62.1$ mm

Taking larger of the two values, we have $d = 62.1$ mm **Ans.**

Problem:

A 50 mm diameter shaft is made from carbon steel having ultimate tensile strength of 630 MPa. It is subjected to a torque which fluctuates between 2000 N-m to – 800 N-m. Using Soderberg method, calculate the factor of safety. Assume suitable values for any other data needed.

Solution. Given: $d = 50$ mm ; $\sigma_u = 630$ MPa $= 630$ N/mm^2 ; $T_{max} = 2000$ N-m ; $T_{min} = -800$ N-m

We know that the mean or average torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{2000 + (-800)}{2} = 600 \text{ N-m} = 600 \times 10^3 \text{ N-mm}$$

$\therefore$ Mean or average shear stress,

$$\tau_m = \frac{16\, T_m}{\pi d^3} = \frac{16 \times 600 \times 10^3}{\pi (50)^3} = 24.4 \text{ N/mm}^2 \quad \ldots\left(\because\ T = \frac{\pi}{16} \times \tau \times d^3\right)$$

Variable torque,

$$T_v = \frac{T_{max} - T_{min}}{2} = \frac{2000 - (-800)}{2} = 1400 \text{ N-m} = 1400 \times 10^3 \text{ N-mm}$$

$\therefore$ Variable shear stress, $\tau_v = \dfrac{16\, T_v}{\pi d^3} = \dfrac{16 \times 1400 \times 10^3}{\pi (50)^3} = 57 \text{ N/mm}^2$

Since the endurance limit in reversed bending (σ_e) is taken as one-half the ultimate tensile strength (*i.e.* $\sigma_e = 0.5\,\sigma_u$) and the endurance limit in shear (τ_e) is taken as $0.55\,\sigma_e$, therefore

$$\tau_e = 0.55\,\sigma_e = 0.55 \times 0.5\,\sigma_u = 0.275\,\sigma_u$$
$$= 0.275 \times 630 = 173.25 \text{ N/mm}^2$$

Assume the yield stress (σ_y) for carbon steel in reversed bending as 510 N/mm^2, surface finish factor (K_{sur}) as 0.87, size factor (K_{sz}) as 0.85 and fatigue stress concentration factor (K_{fs}) as 1.

Since the yield stress in shear (τ_y) for shear loading is taken as one-half the yield stress in reversed bending (σ_y), therefore

$$\tau_y = 0.5\,\sigma_y = 0.5 \times 510 = 255 \text{ N/mm}^2$$

Let $F.S.$ = Factor of safety.

We know that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} = \frac{24.4}{255} + \frac{57 \times 1}{173.25 \times 0.87 \times 0.85}$$
$$= 0.096 + 0.445 = 0.541$$

$\therefore$ $F.S. = 1 / 0.541 = 1.85$ **Ans.**

Problem

A simply supported beam has a concentrated load at the centre which fluctuates from a value of P to 4 P. The span of the beam is 500 mm and its cross-section is circular with a diameter of 60 mm. Taking for the beam material an ultimate stress of 700 MPa, a yield stress of 500 MPa, endurance limit of 330 MPa for reversed bending, and a factor of safety of 1.3, calculate the maximum value of P. Take a size factor of 0.85 and a surface finish factor of 0.9.

Solution. Given : $W_{min} = P$; $W_{max} = 4P$; $L = 500$ mm; $d = 60$ mm ; $\sigma_u = 700$ MPa = 700 N/mm^2 ; $\sigma_y = 500$ MPa = 500 N/mm^2 ; $\sigma_e = 330$ MPa = 330 N/mm^2 ; $F.S. = 1.3$; $K_{sz} = 0.85$; $K_{sur} = 0.9$

We know that maximum bending moment,

$$M_{max} = \frac{W_{max} \times L}{4} = \frac{4P \times 500}{4} = 500P \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times L}{4} = \frac{P \times 500}{4} = 125P \text{ N mm}$$

∴ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{500P + 125P}{2} = 312.5P \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{500P - 125P}{2} = 187.5P \text{ N-mm}$$

Section modulus, $$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(60)^3 = 21.21 \times 10^3 \text{ mm}^3$$

∴ Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{312.5\,P}{21.21 \times 10^3} = 0.0147\,P \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{187.5\,P}{21.21 \times 10^3} = 0.0088\,P \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147\,P}{700} + \frac{0.0088\,P \times 1}{330 \times 0.9 \times 0.85} \qquad \text{...(Taking } K_f = 1)$$

$$= \frac{21P}{10^6} + \frac{34.8\,P}{10^6} = \frac{55.8\,P}{10^6}$$

$$\therefore \qquad P = \frac{1}{1.3} \times \frac{10^6}{55.8} = 13\,785 \text{ N} = 13.785 \text{ kN}$$

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

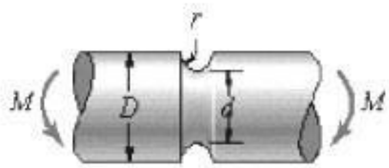
$$\frac{1}{1.3} = \frac{0.0147\,P}{500} + \frac{0.0088\,P \times 1}{330 \times 0.9 \times 0.85} = \frac{29.4\,P}{10^6} + \frac{34.8\,P}{10^6} = \frac{64.2P}{10^6}$$

$$\therefore \qquad P = \frac{1}{1.3} \times \frac{10^6}{64.2} = 11\,982 \text{ N} = 11.982 \text{ kN}$$

From the above, we find that maximum value of P = 13.785 kN **Ans.**

Problem

A grooved shaft shown in **figure- 3.4.4.1** is subjected to rotating-bending load. The dimensions are shown in the figure and the bending moment is 30 Nm. The shaft has a ground finish and an ultimate tensile strength of 1000 MPa. Determine the life of the shaft.



r = 0.4 mm
D = 12 mm
d = 10 mm

3.4.4.1F

Modified endurance limit, $\sigma_e' = \sigma_e\, C_1 C_2 C_3 C_4 C_5 / K_f$

Here, the diameter lies between 7.6 mm and 50 mm : C_1 = 0.85

The shaft is subjected to reversed bending load: C_2 = 1

From the surface factor vs tensile strength plot in **figure- 3.3.3.5**

From the notch sensitivity plots in **figure- 3.3.4.2** , for r=0.4 mm and UTS = 1000 MPa, q = 0.78

From stress concentration plots in **figure-3.4.4.2**, for r/d = 0.04 and D/d = 1.2, K_t = 1.9. This gives $K_f = 1+q(K_t-1) = 1.702$.

Then, $\sigma_e' = \sigma_e x\ 0.89 x\ 1 x\ 0.91 x\ 1 x\ 0.702/1.702 = 0.319\ \sigma_e$

For steel, we may take $\sigma_e = 0.5\ \sigma_{UTS}$ = 500 MPa and then we have

σ_e' = 159.5 MPa.

Bending stress at the outermost fiber, $\sigma_b = \frac{32M}{\pi d^3}$

For the smaller diameter, d=0.01 mm, $\sigma_b = 305$ MPa

Since $\sigma_b > \sigma_e^{'}$ life is finite.

For high cycle fatigue with finite life,

log S = b log N + C

where, $b = -\frac{1}{3}\log\frac{0.8\sigma_0}{\sigma_e'} = -\frac{1}{3}\log\frac{0.8x1000}{159.5} = -0.233$

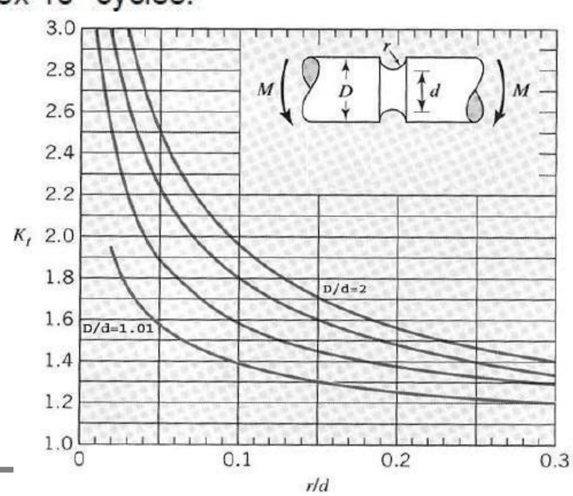
$$c = \log\frac{(0.8\sigma_u)^2}{\sigma_e'} = \log\frac{(0.8x1000)^2}{159.5} = 3.60$$

Therefore, finite life N can be given by

$N=10^{-c/b}\ S^{1/b}$ if $10^3 \le N \le 10^6$.

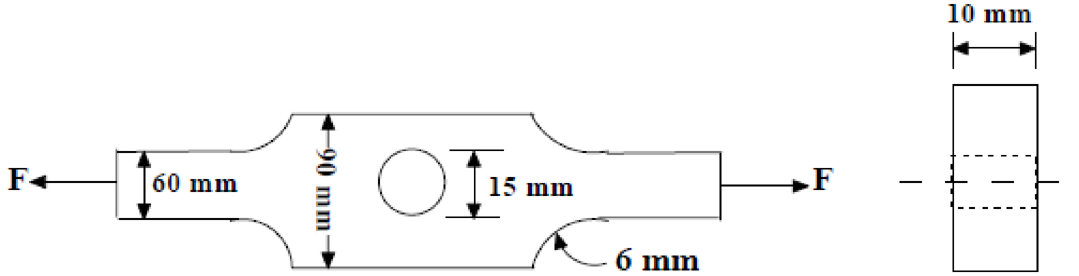
Since the reversed bending stress is 306 MPa,

N = 2.98x 10^9 cycles.



3.4.4.2.F

A portion of a connecting link made of steel is shown in **figure-3.4.4.3** . The tensile axial force F fluctuates between 15 KN to 60 KN. Find the factor of safety if the ultimate tensile strength and yield strength for the material are 440 MPa and 370 MPa respectively and the component has a machine finish.



3.4.4.3F

To determine the modified endurance limit at the step, $\sigma_e' = \sigma_e C_1C_2C_3C_4C_5/ K_f$ where

C_1 = 0.75 since d $\geq$ 50 mm

C_2 = 0.85 for axial loading

C_3 = 0.78 since σ_u = 440 MPa and the surface is machined.

C_4 = 1 since T$\leq$ 450°C

C_5 = 0.75 for high reliability.

At the step, r/d = 0.1, D/d = 1.5 and from **figure-3.2.4.6**, K_t = 2.1 and from **figure- 3.3.4.2** q = 0.8. This gives $K_f = 1+q\,(K_t-1) = 1.88$.

Modified endurance limit, $\sigma_e' = \sigma_e$x 0.75x 0.85x 0.82x 1x 0.75/1.88 = 0.208 σ_e

Take $\sigma_e = 0.5\,\sigma_u$. Then σ_e' = 45.76 MPa.

The link is subjected to reversed axial loading between 15 KN to 60 KN.

This gives $\sigma_{max} = \dfrac{60x10^3}{0.01x0.06} = 100\,MPa$, $\sigma_{min} = \dfrac{15x10^3}{0.01x0.06} = 25\,MPa$

Therefore, σ_{mean} = 62.5 MPa and σ_v = 37.5 MPa.

Using Soderberg's equation we now have,

$\dfrac{1}{F.S} = \dfrac{62.5}{370} + \dfrac{37.5}{45.75}$ so that F.S = 1.011

This is a low factor of safety.

Consider now the endurance limit modification at the hole. The endurance limit modifying factors remain the same except that K_f is different since K_t is different. From **figure- 3.2.4.7** for d/w= 15/90 = 0.25, K_t = 2.46 and q remaining the same as before i.e 0.8

Therefore, $K_f = 1+q\,(K_t-1) = 2.163$.

This gives $\sigma_e' = 39.68$ MPa. Repeating the calculations for F.S using Soderberg's equation , F.S = 0.897.

This indicates that the plate may fail near the hole.

Problem

A 60 mm diameter cold drawn steel bar is subjected to a completely reversed torque of 100 Nm and an applied bending moment that varies between 400 Nm and -200 Nm. The shaft has a machined finish and has a 6 mm diameter hole drilled transversely through it. If the ultimate tensile stress σ_u and yield stress σ_y of the material are 600 MPa and 420 MPa respectively, find the factor of safety.

The mean and fluctuating torsional shear stresses are

$$\tau_m = 0\ ;\ \tau_v = \frac{16x100}{\pi x\left(0.06\right)^3} = 2.36\ \text{MPa}.$$

and the mean and fluctuating bending stresses are

$$\sigma_m = \frac{32x100}{\pi x\left(0.06\right)^3} = 4.72\ \text{MPa};\quad \sigma_v = \frac{32x300}{\pi x\left(0.06\right)^3} = 14.16\ \text{MPa}.$$

For finding the modifies endurance limit we have,

C_1 = 0.75 since d > 50 mm

C_2 = 0.78 for torsional load

= 1 for bending load

C_3 = 0.78 since σ_u = 600 MPa and the surface is machined (**figure- 3.4.4.2**).

C_4 = 1 since $T \leq 450^{\circ}C$

C_5 = 0.7 for high reliability.

and K_f = 2.25 for bending with d/D =0.1 (from **figure- 3.4.4.5**)

= 2.9 for torsion on the shaft surface with d/D = 0.1 (from **figure- 3.4.4.6**)

This gives for bending $\sigma_{eb}' = \sigma_e \times 0.75 \times 1 \times 0.78 \times 1 \times 0.7/2.25 = 0.182\ \sigma_e$

For torsion $\sigma_{es}' = \sigma_{es} \times 0.75 \times 0.78 \times 0.78 \times 1 \times 0.7/2.9 = 0.11\ \sigma_e$

And if $\sigma_e = 0.5\ \sigma_u$ = 300 MPa, σ_{eb}' =54.6 MPa; σ_{es}' = 33 MPa

We may now find the equivalent bending and torsional shear stresses as:

$$\tau_{eq} = \tau_m + \tau_v \frac{\tau_y}{\sigma_{es}'} = 15.01 \text{ MPa} \quad (\text{Taking } \tau_y = 0.5\ \sigma_y = 210 \text{ MPa})$$

$$\sigma_{eq} = \sigma_m + \sigma_v \frac{\sigma_y}{\sigma_{eb}'} = 113.64 \text{ MPa.}$$

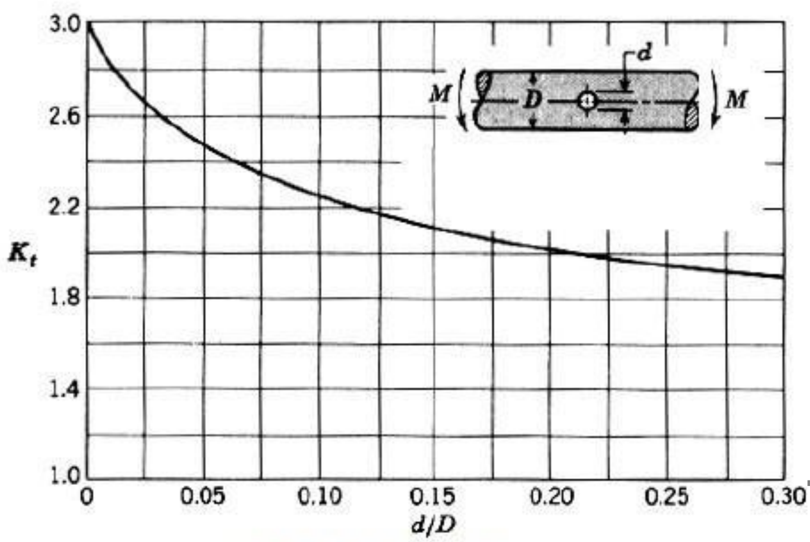
Equivalent principal stresses may now be found as

$$\sigma_{1eq} = \frac{\sigma_{eq}}{2} + \sqrt{\left(\frac{\sigma_{eq}}{2}\right)^2 + \tau_{eq}^2}$$

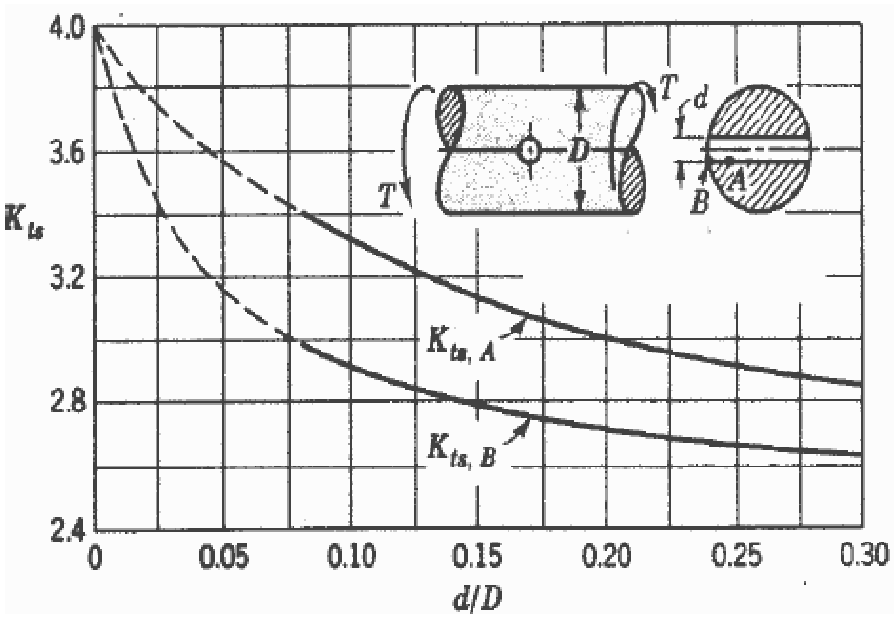
$$\sigma_{2eq} = \frac{\sigma_{eq}}{2} - \sqrt{\left(\frac{\sigma_{eq}}{2}\right)^2 + \tau_{eq}^2}$$

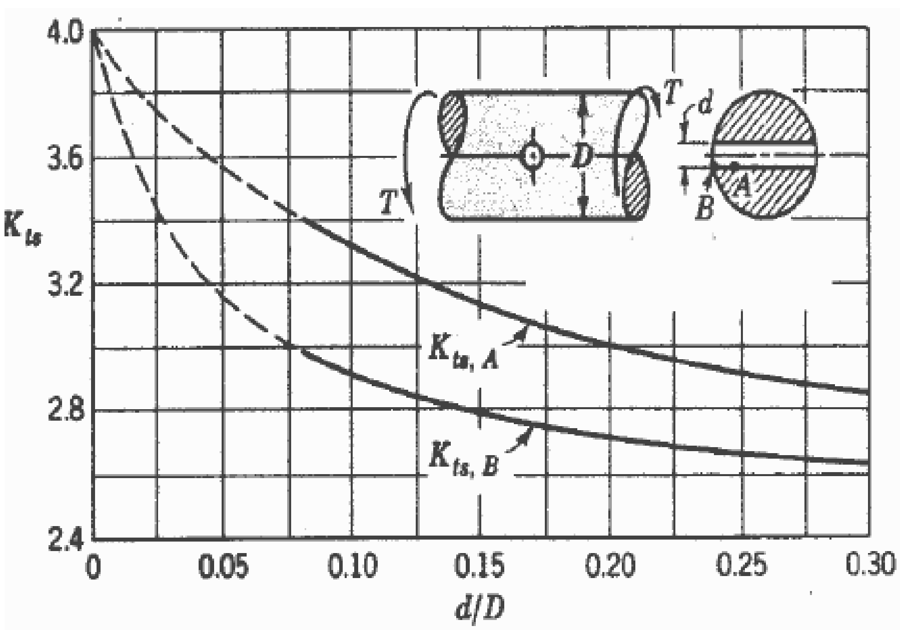
and using von-Mises criterion

$$\sigma_{eq}^2 + 3\tau_{eq}^2 = 2\left(\frac{\sigma_y}{F.S}\right)^2 \quad \text{which gives F.S = 5.18.}$$



3.4.4.5 F (Ref.[2])


4.0
3.6
3.2
2.8
2.4
K_{ts}
T
d
D
A
B
$K_{ts,A}$
$K_{ts,B}$
0
0.05
0.10
0.15
0.20
0.25
0.30
d/D


4.0
3.6
3.2
2.8
2.4
K_{ts}
T
d
D
A
B
$K_{ts,A}$
$K_{ts,B}$
0
0.05
0.10
0.15
0.20
0.25
0.30
d/D

3.4.4.6 F (Ref.[2])

UNIT 4

THREADED FASTENERS

Instructional Objectives

- *Some details of tapping and set screws.*
- *Thread forms in details.*
- *Different types of stresses developed in screw fasteners due to initial tightening and external load.*
- *Combined effect of initial tightening and external load on a bolted joint.*

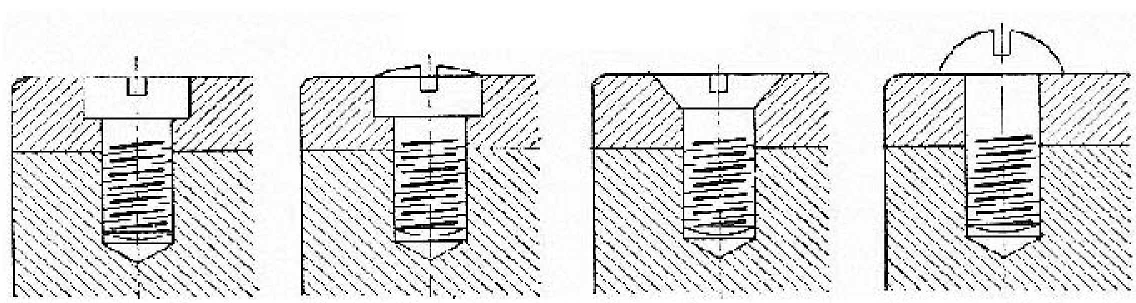
Bolts, screws and studs are the most common types of threaded fasteners. They are used in both permanent and removable joints.

Bolts: They are basically threaded fasteners normally used with nuts.

Screws: They engage either with preformed or self made internal threads.

Studs: They are externally threaded headless fasteners. One end usually meets a tapped component and the other with a standard nut.

There are different forms of bolt and screw heads for a different usage. These include bolt heads of square, hexagonal or eye shape and screw heads of hexagonal, Fillister, button head, counter sunk or Phillips type. These are shown in figures.



Flat Fillister Head — Oval Fillister Head — Flat or Countersunk Head — Round or Button Head

Fig: Types of screw heads

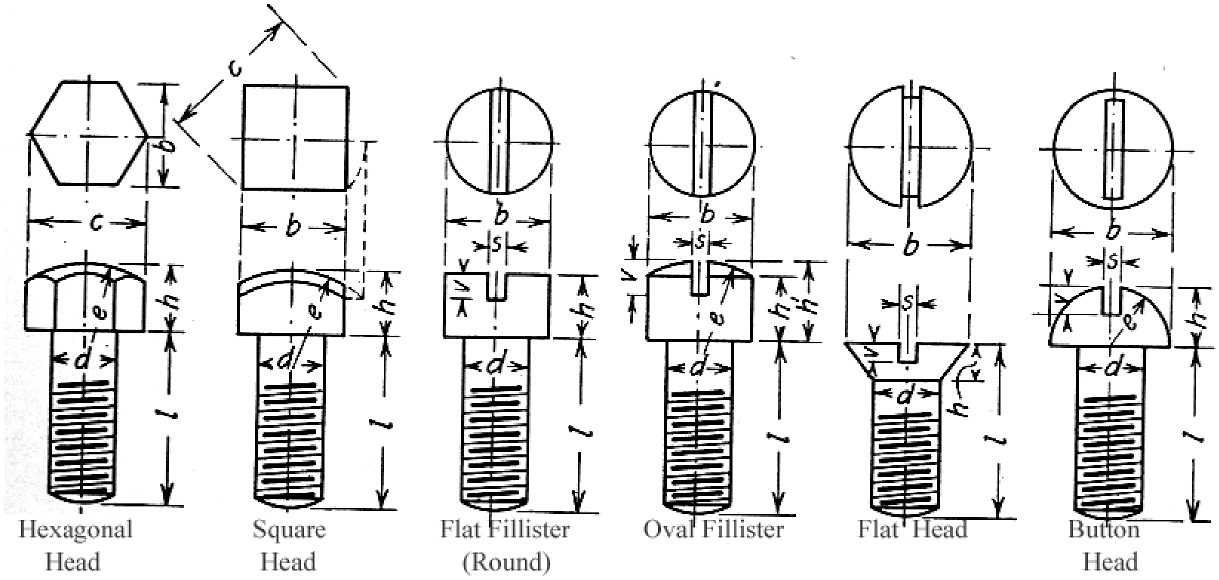


Fig: Types of bolt heads.

Tapping screws

These are one piece fasteners which cut or form a mating thread when driven into a preformed hole. These allow rapid installation since nuts are not used. There are two types of tapping screws. They are known as thread forming which displaces or forms the adjacent materials and thread cutting which have cutting edges and chip cavities which create a mating thread.

Set Screws

These are semi permanent fasteners which hold collars, pulleys, gears etc on a shaft. Different heads and point styles are available.

Thread forms

Basically when a helical groove is cut or generated over a cylindrical or conical section, threads are formed. When a point moves parallel to the axis of a rotating cylinder or cone held between centers, a helix is generated. Screw threads formed in this way have two functions to perform in general:

(a) To transmit power - Square. ACME, Buttress, Knuckle types of thread forms is useful for this purpose.

(b) To secure one member to another- V-threads are most useful for this purpose. Some standard forms are shown in figure**.**

V-threads are generally used for securing because they do not shake loose due to the wedging action provided by the thread. Square threads give higher efficiency due to a low friction. This is demonstrated in figure.

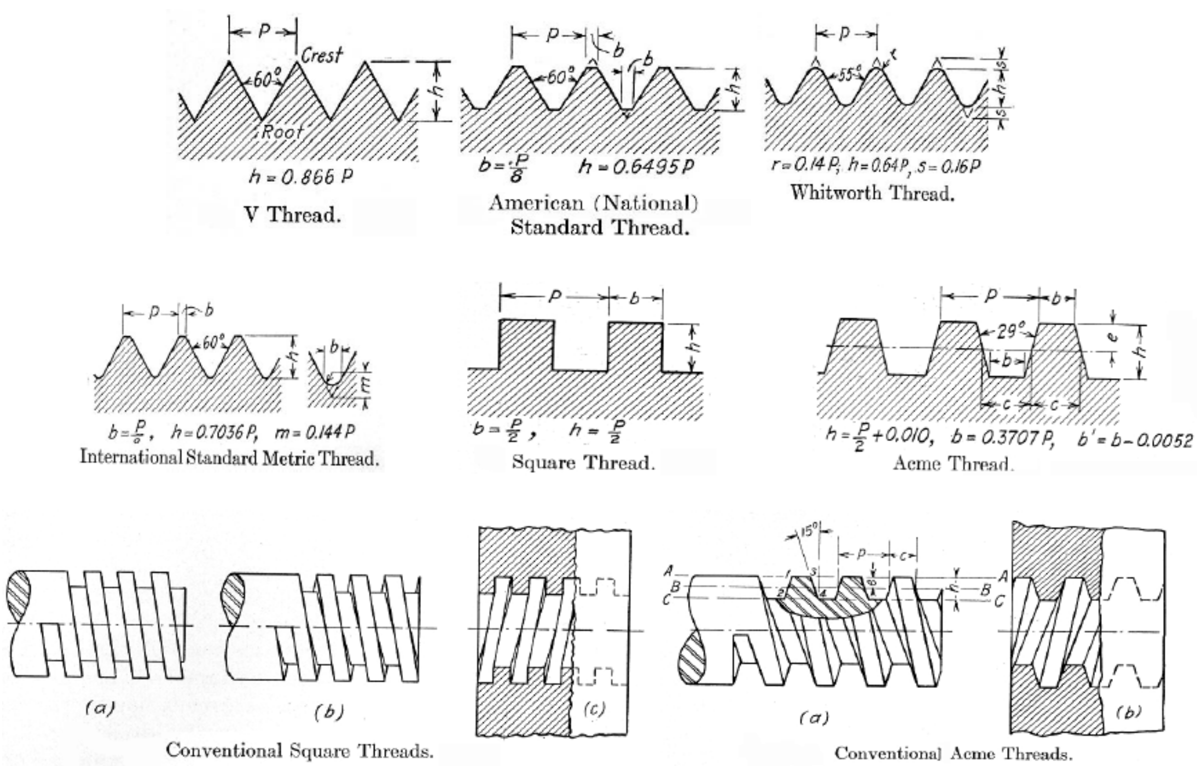
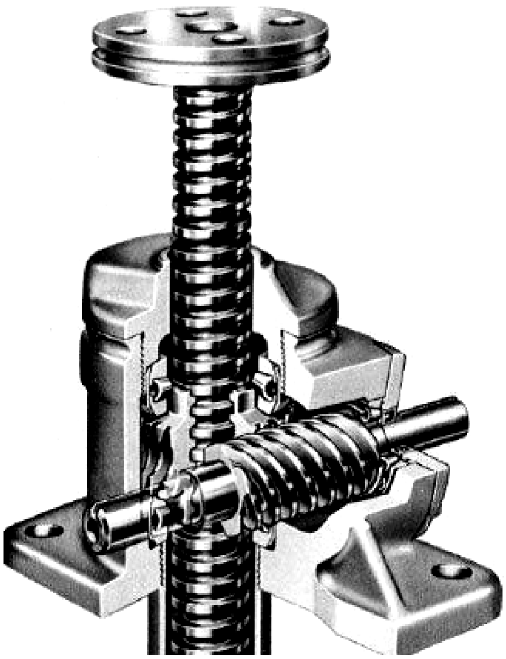
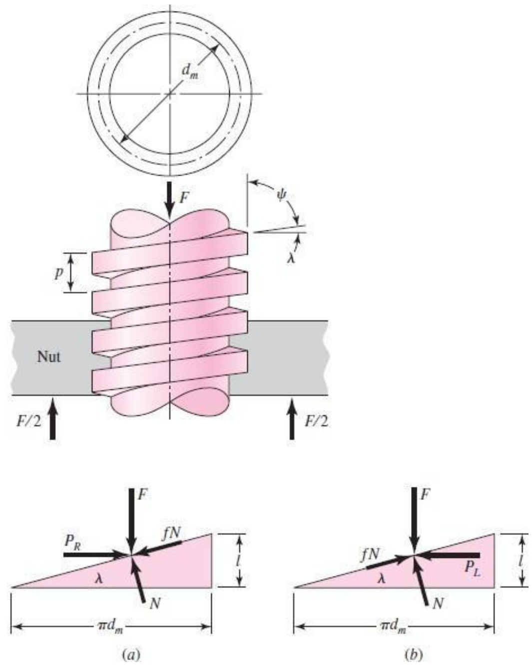


Fig: Different types of thread forms.

The Mechanics of Power Screws

A power screw is a device used in machinery to change angular motion into linear motion, and, usually, to transmit power. Familiar applications include the lead screws of lathes, and the screws for vises, presses, and jacks. An application of power screws to a power-driven jack is shown. You should be able to identify the worm, the worm gear, the screw, and the nut. In a square-threaded power screw with single thread having a mean diameter *dm*, a pitch *p*, a lead angle λ, and a helix angle ψ is loaded by the axial compressive force *F*. We wish to find an expression for the torque required to raise this load, and another expression for the torque required to lower the load. First, imagine that a single thread of the screw is unrolled or developed for exactly a single turn. Then one edge of the thread will form the hypotenuse of a right triangle whose base is the circumference of the mean-thread-diameter circle and whose height is the lead. The angle λ is the lead angle of the thread. We represent the summation of all the unit axial forces acting upon the normal thread area by *F*. To raise the load, a force *PR* acts to the righ, and to lower the load, *PL* acts to the left. The friction force is the product of the coefficient of friction *f* with the normal force *N*, and acts to oppose the motion. The system is in equilibrium under the action of these forces, and hence, for raising the load, we have





$$\sum F_H = P_R - N \sin\lambda - f N \cos\lambda = 0 \qquad (a)$$

$$\sum F_V = F + f N \sin\lambda - N \cos\lambda = 0$$

In a similar manner, for lowering the load, we have

$$\sum F_H = -P_L - N \sin\lambda + f N \cos\lambda = 0 \qquad (b)$$

$$\sum F_V = F - f N \sin\lambda - N \cos\lambda = 0$$

Since we are not interested in the normal force *N*, we eliminate it from each of these sets of equations and solve the result for *P*. For raising the load, this gives

$$P_R = \frac{F(\sin\lambda + f\cos\lambda)}{\cos\lambda - f\sin\lambda} \qquad (c)$$

and for lowering the load,

$$P_L = \frac{F(f\cos\lambda - \sin\lambda)}{\cos\lambda + f\sin\lambda} \tag{d}$$

Next, divide the numerator and the denominator of these equations by $\cos\lambda$ and use the relation $\tan\lambda = l/\pi d_m$ (Fig. 8–6). We then have, respectively,

$$P_R = \frac{F[(l/\pi d_m) + f]}{1 - (fl/\pi d_m)} \tag{e}$$

$$P_L = \frac{F[f - (l/\pi d_m)]}{1 + (fl/\pi d_m)} \tag{f}$$

Finally, noting that the torque is the product of the force P and the mean radius $d_m/2$, for raising the load we can write

$$T_R = \frac{F d_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - fl}\right) \tag{8–1}$$

where T_R is the torque required for two purposes: to overcome thread friction and to raise the load.

The torque required to lower the load, from Eq. (f), is found to be

$$T_L = \frac{F d_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + fl}\right) \tag{8–2}$$

This is the torque required to overcome a part of the friction in lowering the load. It may turn out, in specific instances where the lead is large or the friction is low, that the load will lower itself by causing the screw to spin without any external effort. In such cases, the torque T_L from Eq. (8–2) will be negative or zero. When a positive torque is obtained from this equation, the screw is said to be *self-locking*. Thus the condition for self-locking is

$$\pi f d_m > l$$

Now divide both sides of this inequality by πd_m. Recognizing that $l/\pi d_m = \tan\lambda$, we get

$$f > \tan\lambda \tag{8–3}$$

This relation states that self-locking is obtained whenever the coefficient of thread friction is equal to or greater than the tangent of the thread lead angle.

which, since thread friction has been eliminated, is the torque required only to raise the load. The efficiency is therefore

$$e = \frac{T_0}{T_R} = \frac{Fl}{2\pi T_R} \tag{8–4}$$

The preceding equations have been developed for square threads where the normal thread loads are parallel to the axis of the screw. In the case of Acme or other threads, the normal thread load is inclined to the axis because of the thread angle 2α and the lead angle λ. Since lead angles are small, this inclination can be neglected and only the effect of the thread angle (Fig. 8–7*a*) considered. The effect of the angle α is to increase the frictional force by the wedging action of the threads. Therefore the frictional terms in Eq. (8–1) must be divided by $\cos\alpha$. For raising the load, or for tightening a screw or bolt, this yields

$$T_R = \frac{F d_m}{2}\left(\frac{l + \pi f d_m \sec\alpha}{\pi d_m - f l \sec\alpha}\right) \tag{8–5}$$

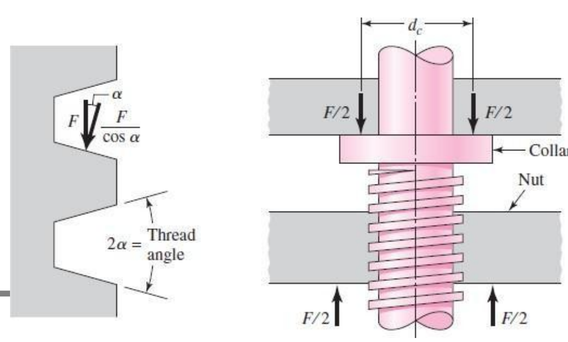
In using Eq. (8–5), remember that it is an approximation because the effect of the lead angle has been neglected.

For power screws, the Acme thread is not as efficient as the square thread, because of the additional friction due to the wedging action, but it is often preferred because it is easier to machine and permits the use of a split nut, which can be adjusted to take up for wear.

Usually a third component of torque must be applied in power-screw applications. When the screw is loaded axially, a thrust or collar bearing must be employed between the rotating and stationary members in order to carry the axial component. Figure 8–7*b* shows a typical thrust collar in which the load is assumed to be concentrated at the mean collar diameter d_c. If f_c is the coefficient of collar friction, the torque required is

$$T_c = \frac{F f_c d_c}{2} \tag{8–6}$$

For large collars, the torque should probably be computed in a manner similar to that employed for disk clutches.



Nominal body stresses in power screws can be related to thread parameters as follows. The maximum nominal shear stress τ in torsion of the screw body can be expressed as

$$\tau = \frac{16T}{\pi d_r^3} \tag{8–7}$$

The axial stress σ in the body of the screw due to load F is

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d_r^2} \tag{8–8}$$

in the absence of column action. For a short column the J. B. Johnson buckling formula is given by Eq. (4–43), which is

$$\left(\frac{F}{A}\right)_{\text{crit}} = S_y - \left(\frac{S_y}{2\pi}\frac{l}{k}\right)^2 \frac{1}{CE} \tag{8–9}$$

Nominal thread stresses in power screws can be related to thread parameters as follows. The bearing stress in Fig. 8–8, σ_B, is

$$\sigma_B = -\frac{F}{\pi d_m n_t p/2} = -\frac{2F}{\pi d_m n_t p} \tag{8–10}$$

where n_t is the number of engaged threads. The bending stress at the root of the thread σ_b is found from

$$\frac{I}{c} = \frac{(\pi d_r n_t)(p/2)^2}{6} = \frac{\pi}{24} d_r n_t p^2 \qquad M = \frac{Fp}{4}$$

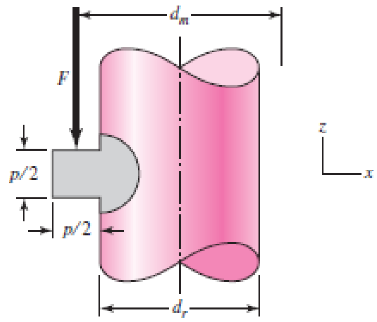
so

$$\sigma_b = \frac{M}{I/c} = \frac{Fp}{4}\frac{24}{\pi d_r n_t p^2} = \frac{6F}{\pi d_r n_t p} \tag{8–11}$$

The transverse shear stress τ at the center of the root of the thread due to load F is

$$\tau = \frac{3V}{2A} = \frac{3}{2}\frac{F}{\pi d_r n_t p/2} = \frac{3F}{\pi d_r n_t p} \tag{8–12}$$

and at the top of the root it is zero. The von Mises stress σ' at the top of the root "plane" is found by first identifying the orthogonal normal stresses and the shear stresses. From



the coordinate system of Fig. 8–8, we note

$$\sigma_x = \frac{6F}{\pi d_r n_t p} \qquad \tau_{xy} = 0$$

$$\sigma_y = 0 \qquad \tau_{yz} = \frac{16T}{\pi d_r^3}$$

$$\sigma_z = -\frac{4F}{\pi d_r^2} \qquad \tau_{zx} = 0$$

then use Eq. (5–14) of Sec. 5–5.

The screw-thread form is complicated from an analysis viewpoint. Remember the origin of the tensile-stress area A_t, which comes from experiment. A power screw lifting a load is in compression and its thread pitch is *shortened* by elastic deformation. Its engaging nut is in tension and its thread pitch is *lengthened*. The engaged threads cannot share the load equally. Some experiments show that the first engaged thread carries 0.38 of the load, the second 0.25, the third 0.18, and the seventh is free of load. In estimating thread stresses by the equations above, substituting $0.38F$ for F and setting n_t to 1 will give the largest level of stresses in the thread-nut combination.

Problem:

A square-thread power screw has a major diameter of 32 mm and a pitch of 4 mm with double threads, and it is to be used in an application similar to that in Fig. 8–4. The given data include $f = f_c = 0.08$, $d_c = 40$ mm, and $F = 6.4$ kN per screw.

(*a*) Find the thread depth, thread width, pitch diameter, minor diameter, and lead.
(*b*) Find the torque required to raise and lower the load.
(*c*) Find the efficiency during lifting the load.
(*d*) Find the body stresses, torsional and compressive.
(*e*) Find the bearing stress.
(*f*) Find the thread stresses bending at the root, shear at the root, and von Mises stress and maximum shear stress at the same location.

(*a*) From Fig. 8–3*a* the thread depth and width are the same and equal to half the pitch, or 2 mm. Also

$$d_m = d - p/2 = 32 - 4/2 = 30 \text{ mm}$$

$$d_r = d - p = 32 - 4 = 28 \text{ mm}$$

$$l = np = 2(4) = 8 \text{ mm}$$

(*b*) Using Eqs. (8–1) and (8–6), the torque required to turn the screw against the load is

$$T_R = \frac{F d_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - f l}\right) + \frac{F f_c d_c}{2}$$

$$= \frac{6.4(30)}{2}\left[\frac{8 + \pi(0.08)(30)}{\pi(30) - 0.08(8)}\right] + \frac{6.4(0.08)40}{2}$$

$$= 15.94 + 10.24 = 26.18 \text{ N} \cdot \text{m}$$

Using Eqs. (8–2) and (8–6), we find the load-lowering torque is

$$T_L = \frac{F d_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + f l}\right) + \frac{F f_c d_c}{2}$$

$$= \frac{6.4(30)}{2}\left[\frac{\pi(0.08)30 - 8}{\pi(30) + 0.08(8)}\right] + \frac{6.4(0.08)(40)}{2}$$

$$= -0.466 + 10.24 = 9.77 \text{ N} \cdot \text{m}$$

The minus sign in the first term indicates that the screw alone is not self-locking and would rotate under the action of the load except for the fact that the collar friction is present and must be overcome, too. Thus the torque required to rotate the screw "with" the load is less than is necessary to overcome collar friction alone.

(*c*) The overall efficiency in raising the load is

$$e = \frac{Fl}{2\pi T_R} = \frac{6.4(8)}{2\pi(26.18)} = 0.311$$

(*d*) The body shear stress τ due to torsional moment T_R at the outside of the screw body is

$$\tau = \frac{16T_R}{\pi d_r^3} = \frac{16(26.18)(10^3)}{\pi(28^3)} = 6.07 \text{ MPa}$$

The axial nominal normal stress σ is

$$\sigma = -\frac{4F}{\pi d_r^2} = -\frac{4(6.4)10^3}{\pi(28^2)} = -10.39 \text{ MPa}$$

(*e*) The bearing stress σ_B is, with one thread carrying $0.38F$,

$$\sigma_B = -\frac{2(0.38F)}{\pi d_m(1)p} = -\frac{2(0.38)(6.4)10^3}{\pi(30)(1)(4)} = -12.9 \text{ MPa}$$

(*f*) The thread-root bending stress σ_b with one thread carrying $0.38F$ is

$$\sigma_b = \frac{6(0.38F)}{\pi d_r(1)p} = \frac{6(0.38)(6.4)10^3}{\pi(28)(1)4} = 41.5 \text{ MPa}$$

The transverse shear at the extreme of the root cross section due to bending is zero. However, there is a circumferential shear stress at the extreme of the root cross section of the thread as shown in part (*d*) of 6.07 MPa. The three-dimensional stresses, after Fig. 8–8, noting the y coordinate is into the page, are

$$\sigma_x = 41.5 \text{ MPa} \qquad \tau_{xy} = 0$$
$$\sigma_y = 0 \qquad \tau_{yz} = 6.07 \text{ MPa}$$
$$\sigma_z = -10.39 \text{ MPa} \qquad \tau_{zx} = 0$$

Equation (5–14) of Sec. 5–5 can be written as

$$\sigma' = \frac{1}{\sqrt{2}}\{(41.5-0)^2 + [0-(-10.39)]^2 + (-10.39-41.5)^2 + 6(6.07)^2\}^{1/2}$$
$$= 48.7 \text{ MPa}$$

Alternatively, you can determine the principal stresses and then use Eq. (5–12) to find the von Mises stress. This would prove helpful in evaluating τ_{max} as well. The principal stresses can be found from Eq. (3–15); however, sketch the stress element and note that there are no shear stresses on the x face. This means that σ_x is a principal stress. The remaining stresses can be transformed by using the plane stress equation, Eq. (3–13). Thus, the remaining principal stresses are

$$\frac{-10.39}{2} \pm \sqrt{\left(\frac{-10.39}{2}\right)^2 + 6.07^2} = 2.79, -13.18 \text{ MPa}$$

Ordering the principal stresses gives $\sigma_1, \sigma_2, \sigma_3 = 41.5, 2.79, -13.18$ MPa. Substituting these into Eq. (5–12) yields

Ham and Ryan1 showed that the coefficient of friction in screw threads is independent of axial load, practically independent of speed, decreases with heavier lubricants, shows little variation with combinations of materials, and is best for steel on bronze. Sliding coefficients of friction in power screws are about 0.10–0.15.

Problem:

An M20x2 steel bolt of 100mm long is subjected to impact load. The energy absorbed by the bolt is 2 N-m,

i) Determine the stress in the shank of the bolt if there is no threaded portion between the nut and bolt head.

ii) Determine the stress in the shank if the entire length of the bolt is threaded.

Assume modulus of elasticity for steel as 206 GPa.

Given $M_{20} \times 2$ L = 100mm, impact load, U = 2 Nm = 2×10^3N-mm

From DDHB care diameter d = 17.546mm

$$\text{Care Area} = A_c = \frac{\pi}{4} \times 17.546^2 = 241.63\text{mm}^2$$

$$\text{Outer Area} = A_s = \frac{\pi}{4} \times 20^2 = 314.16\text{mm}^2$$

$$E = 206 \times 10^3 \text{N/mm}^2$$

$E = 206 \times 10^3$ N/mm^2

(i) Case i : Where there are no threads:

$$\text{Impact energy } U = \frac{1}{2} \times F \times \zeta \qquad \text{But } \zeta = \frac{FL}{AE}$$

$$2 \times 10^3 = \frac{1}{2} \times F \times \frac{F \times 100}{314.16 \times 206 \times 10^3}$$

$$\therefore F = 50879\text{N}$$

$$\therefore \text{Impact stress} = 50879 = 161.95 \text{ N/mm}^2$$

(i) Case ii : Threaded region

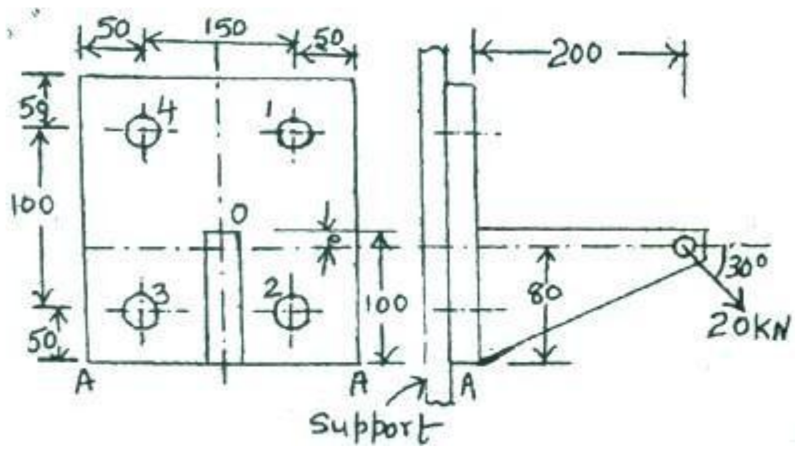
$$2 \times 10^3 = \frac{1}{2} \times F \times \frac{F \times 100}{241.63 \times 206 \times 10^3}$$

$$\therefore F - 44.620\text{KN}$$

$$\text{Impact stress} = \frac{\text{Impact Load}}{\text{Area}} = \frac{44620}{241.63} = 184.6 \text{ N/mm}^2$$

Determine the size of the bolts for the loaded brackets shown in Fig, if the allowable

Tensile stress in the bolt material is limited to 80 MPa.



Horizontal force component

$F_H = F\cos 30$

$F_H = 20000 \times \cos 30 = 17320N$

$F_V = F\sin 30$

$F_H = 20000 \times \sin 30 = 10000N$

i) Effect of vertical load :

a) Direct shear force on each bolt

b) Turning moment about the tilting edge in CW.

$$a)\ \frac{10000}{4} = 2500N$$

$$b)\ M_{t_v} = F_v \times 200 = 10000 \times 200 = 2 \times 10^6\,N-mm$$

ii) Effect of horizontal load:

a) Direct tensile force

b) Turning moment about the CG in CCW.

$$a)\ \frac{F_H}{4} = \frac{17320}{4} = 4330N$$

$$b)\ M_{t_H} = F_H \times 20 = 17320 \times 20 = 346400N-mm = 3.4 \times 10^5\,N-mm$$

$$M_{t_H} = -3.4 \times 10^5 \text{ because of CCW}$$

$$\therefore \text{Resultant Momentum}: M_t = M_{tH} + M_{tv}$$

$$= 2 \times 10^6 - 3.4 \times 10^5 = 1.66 \times 10^6\,N-mm$$

$$\therefore \text{Secondary tensile force } F_{t_2} = \frac{M_t L_1}{L_1^2 + L_2^2 + L_3^2 + L_4^2}$$

$$= \frac{1.66 \times 10^6 \times 150}{150^2 + 150^2 + 50^2 + 50^2} = 4980N$$

$\therefore$ Total tensile force $= 4330 + 4980 = 9310\text{N}$

$$\text{Max shear force } F_{s_{max}} = \sqrt{\left(\frac{F_t}{2}\right)^2 + F_s^2} = \sqrt{\left(\frac{9310}{2}\right)^2 + 2500^2} = 5283.8\text{N}$$

iii) Size of bolts:

$F_{s\,max} = \tau_{max} \times A_c$

$5283.8 = 40 \times A_c$

$A_c = 132.1\,\text{mm}^2$

$d_c = 12.96\text{mm}$

majan $\phi = d_c / 0.84 = 15.42\,\text{mm}$

M16

Problem:

A MIO steel bolt of 125 mm- long is subjected to an impact load. The kinetic energy absorbed by the bolt is 2.5 Joules. Determine (i) Stress in the shank of the bolt if there is no threaded portion between the nut and the bolt head, (ii) Stress in the shank of the area of the shank is reduced to that of the root area of the threaded or the entire length of the bolt is threaded.

T 18.7 core dia of M10 bolt d_1=8.159696 mm

core area $A_c = \frac{\pi}{4}(8.1596)^2 = 52.292\text{ mm}^2$

Shank area $A = \frac{\pi}{4}(10)^2 = 78.54\text{ mm}^2$

$E = 206.88\text{ GPa} = 206.88 \times 10^3\text{ MPa}$

T 2.8/2.10

(a) Stress in the shank (No thread between bolt head and the nut)

energy of impact will be absorbed, Impact energy $\Rightarrow U = \frac{1}{2}F\delta = \frac{F}{2}\left(\frac{FL}{AE}\right)$

$$2.5 \times 10^3 = \frac{F^2\,(125)}{2 \times 78.54 \times 206.8 \times 10^3}$$

$$\text{Impact Stress} \Rightarrow \sigma^1 = \frac{\text{Impact load}}{\text{core area}} = \frac{25488.88}{52.292} = 487.4336 \text{ MPa}$$

(b) Stress in the shank area, if the shank area is reduced to that of root

$$\text{Impact energy} \Rightarrow U = \frac{1}{2} F\delta = \frac{F}{2}\left(\frac{FL}{AE}\right)$$

$$2.5 \times 10^3 = \frac{F^2(125)}{2 \times 52.292 \times 206.8 \times 10^3} \Rightarrow F = 20798.063\ N$$

$$\text{Impact stress} \Rightarrow \sigma^1 = \frac{F}{A_c} = \frac{20798.063}{52.292} = 397.73 \text{ MPa}$$

Problem:

A bolt is subjected to initial loading of 5 kN and final tensile load of 9 kN. Determine the size of the bolt, if the allowable stress is 80 MPa and K = 0.05.

E18.4 Final load on the bolt $F_f = kF_a + F_i = 0.05(9000) + 5000 = 5450N$

Since $F_a > F_f$ take final load on bolt as F_a, i.e., $F_f = 9000N$

$$\text{Core area of thread } A_c = \frac{F_f}{\sigma} = \frac{9000}{80} = 112.5\text{mm}^2$$

$$\text{and } A_c = \frac{\pi}{4} d_1^2$$

$$\text{Now equating} \quad 112.5 = \frac{\pi}{4} d_1^2 \rightarrow d_1 = 11.968\text{mm}$$

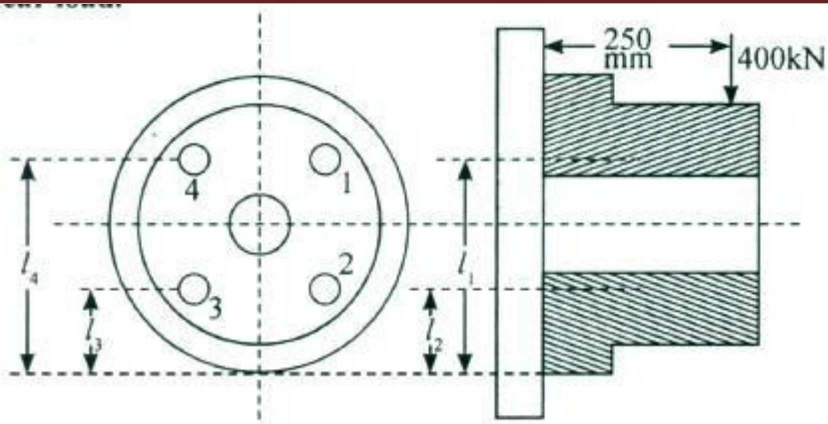
$$\text{major dia } d = \frac{d_1}{0.84} = 14.25\text{mm}$$

From T 18.7 for major Dia d = 14.25mm,

Select coarse thread botl M16

Problem:

A Flange bearing as shown in fig 4(b) is fastened to a fram by means of 4 bolts, spaced equally on a 500mm bolt circle. The dia of the bearing flange in 650mm and a load of 400 kN acts at a distance of 250mm from the frame. Determine the size of bolt, taking allowable tensile stress in the bolt material as 60N/m2. Two dowel pins are provided to take up the shear load.



Given, $i = 4$,

$2b = 500mm, \rightarrow b = 250mm$

$2a = 650mm, \rightarrow a = 325mm$

$F = 400kN = 400 \times 10^3N$

$\sigma = 60\ N/mm^2$

$l = 250mm$

$L_1 = L_4\ \phi\ L_2 = L_3$

Maximum load on tha bolt,

$$F_{max} = \frac{2Fl\left[a + b\cos\left(\frac{180}{i}\right)\right]}{\left(2a^2 + b^2\right)i}$$

$$= \frac{2 \times 400 \times 10^3 \times 250\left[325 + 250\cos\frac{180}{4}\right]}{\left[2 \times 325^2 + 250^2\right] \times 4}$$

$$= 91648.712N$$

Tensile stress area of the bolt,

$$A_c = \frac{F_{max}}{\sigma}$$

$$= \frac{91648.712}{60}$$

$$A_c = 1527.48mm^2$$

From the data hand book, the nearest standard tensile stress area for coarse threads, $A_c = 1760$ mm^2

Hence the bolt selected M52.

UNIT 5

DESIGN OF SHAFTS

Instructional Objectives

- *Definition of shaft*
- *Standard shaft sizes*
- *Standard shaft materials*
- *Design of shaft based on strength*

Introduction

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending. In other words, we may say that a shaft is used for the transmission of torque and bending moment.

The various members are mounted on the shaft by means of keys or splines.

Notes: 1. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

2. An axle, though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.

3. A spindle is a short shaft that imparts motion either to a cutting tool (e.g. drill press spindles) or to a work piece (e.g. lathe spindles).

Material Used for Shafts

The material used for shafts should have the following properties:

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

The mechanical properties of these grades of carbon steel are given in the following table.

Mechanical properties of steels used for shafts

Indian standard designation	*Ultimate tensile strength, MPa*	*Yield strength, MPa*
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

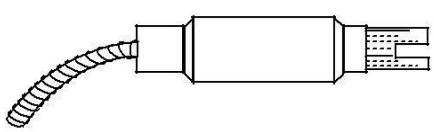
When a shaft of high strength is required, then alloy steel such as nickel, nickel- chromium or chrome-vanadium steel is used.

Types of Shafts

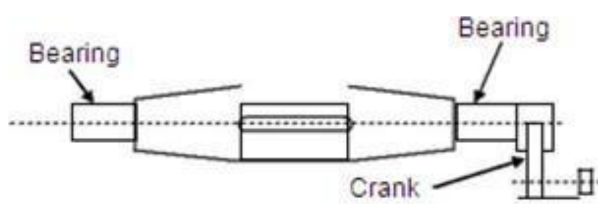
The following two types of shafts are important from the subject point of view:

1. **Transmission shafts.** These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

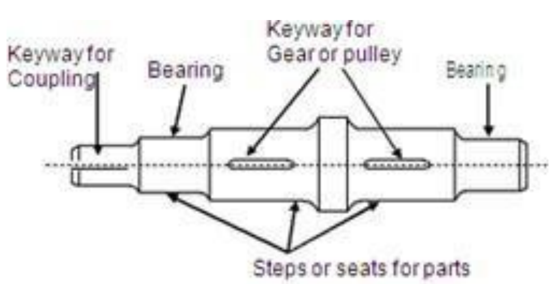
2. **Machine shafts.** These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.



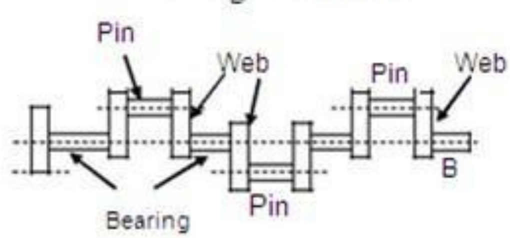
Flexible Shaft



A Single Crank Shaft



A Stepped Shaft



Multiple Crank Shaft

Standard Sizes of Transmission Shafts

The standard sizes of transmission shafts are:

- 25 mm to 60 mm with 5 mm steps
- 60 mm to 110 mm with 10 mm steps
- 110 mm to 140 mm with 15 mm steps
- 140 mm to 500 mm with 20 mm steps.

The standard lengths of the shafts are 5m, 6m and 7m.

Stresses in Shafts

The following stresses are induced in the shafts:

1. Shear stresses due to the transmission of torque (i.e. due to torsional load).
2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
3. Stresses due to combined torsional and bending loads.

Design of Shafts

The shafts may be designed on the basis of

1. Strength and
2. Rigidity and stiffness

In designing shafts on the basis of strength, the following cases may be considered:

(a) Shafts subjected to twisting moment or torque only,

(b) Shafts subjected to bending moment only,

(c) Shafts subjected to combined twisting and bending moments, and

(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Material for Shafts

The ferrous, non-ferrous materials and non metals are used as shaft material depending on the application. Some of the common ferrous materials used for shaft are discussed below.

Hot-rolled plain carbon steel

These materials are least expensive. Since it is hot rolled, scaling is always present on the surface and machining is required to make the surface smooth.

Cold-drawn plain carbon/alloy composition

Since it is cold drawn it has got its inherent characteristics of smooth bright finish. Amount of machining therefore is minimal. Better yield strength is also obtained. This is widely used for general purpose transmission shaft.

Alloy steels

Alloy steel as one can understand is a mixture of various elements with the parent steel to improve certain physical properties. To retain the total advantage of alloying materials one requires heat treatment of the machine components after it has been manufactured. Nickel, chromium and vanadium are some of the common alloying materials. However, alloy steel is expensive.

These materials are used for relatively severe service conditions. When the situation demands great strength then alloy steels are used. They have fewer tendencies to crack, warp or distort in heat treatment. Residual stresses are also less compared to CS (Carbon Steel).

In certain cases the shaft needs to be wear resistant, and then more attention has to be paid to make the surface of the shaft to be wear resistant. The common types of surface hardening methods are,

Hardening of surface

Case hardening and carburizing

Cyaniding and nitriding.

Design considerations for shaft

For the design of shaft following two methods are adopted,

Design based on Strength

In this method, design is carried out so that stress at any location of the shaft should not exceed the material yield stress. However, no consideration for shaft deflection and shaft twist is included.

Design based on Stiffness

Basic idea of design in such case depends on the allowable deflection and twist of the shaft.

Design based on Strength

The stress at any point on the shaft depends on the nature of load acting on it. The stresses which may be present are as follows.

Basic stress equations:

Bending stress,

Where, $$\sigma_b = \frac{32M}{\pi d_0^3(1-k^4)}$$

M : Bending moment at the point of interest

d_o : Outer diameter of the shaft

k : Ratio of inner to outer diameters of the shaft (k = 0 for a solid shaft because inner diameter is zero)

Axial Stress,

Where, $$\sigma_a = \frac{4\alpha F}{\pi d_0^2(1-k^2)}$$

F: Axial force (tensile or compressive)

α: Column-action factor(= 1.0 for tensile load)

The term α has been introduced in the equation. This is known as column action factor. What is a column action factor? This arises due the phenomenon of buckling of long slender members which are acted upon by axial compressive loads.

Here, α is defined as,

$$\alpha = \frac{1}{1-0.0044(L/K)} \quad \text{for } L/K < 115$$

$$\alpha = \frac{\sigma_{yc}}{\pi^2 nE}\left(\frac{L}{K}\right)^2 \quad \text{for } L/K > 115$$

Where,

n = 1.0 for hinged end

n = 2.25 for fixed end

n = 1.6 for ends partly restrained, as in bearing

K = least radius of gyration, L = shaft length

ζ_{yc} = yield stress in compression

Stress due to torsion,

$$\tau_{xy} = \frac{16T}{\pi d_0^3(1-k^4)}$$

Where,

T : Torque on the shaft

η_{xy} : Shear stress due to torsion

Combined Bending and Axial stress

Both bending and axial stresses are normal stresses; hence the net normal stress is given by,

$$\sigma_x = [\frac{32M}{\pi d_0^3(1-k^4)} \pm \frac{4\alpha F}{\pi d_0^2(1-k^2)}]$$

The net normal stress can be either positive or negative. Normally, shear stress due to torsion is only considered in a shaft and shear stress due to load on the shaft is neglected.

Maximum shear stress theory

Design of the shaft mostly uses maximum shear stress theory. It states that a machine member fails when the maximum shear stress at a point exceeds the maximum allowable shear stress for the shaft material. Therefore,

$$\tau_{max} = \tau_{allowable} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

Substituting the values of ζ_x and η_{xy} in the above equation, the final form is,

$$\tau_{allowable} = \frac{16}{\pi d_0^3(1-k^4)}\sqrt{\left\{M + \frac{\alpha F d_0(1+k^2)}{8}\right\}^2 + T^2}$$

Therefore, the shaft diameter can be calculated in terms of external loads and material properties. However, the above equation is further standardized for steel shafting in terms of allowable design stress and load factors in ASME design code for shaft.

ASME design Code

The shafts are normally acted upon by gradual and sudden loads. Hence, the equation is modified in ASME code by suitable load factors,

$$\tau_{allowable} = \frac{16}{\pi d_0^3(1-k^4)}\sqrt{\left\{C_{bm}M + \frac{\alpha F d_0(1+k^2)}{8}\right\}^2 + (C_t T)^2}$$

Where, C_{bm} and C_t are the bending and torsion factors. The values of these factors are given below,

	C_{bm}	C_t
For stationary shaft:		
Load gradually applied	1.0	1.0
Load suddenly applied	1.5 - 2.0	1.5 - 2.0
For rotating shaft:		
Load gradually applied	1.5	1.0
Load suddenly applied (minor shock)	1.5 - 2.0	1.0 - 1.5
Load suddenly applied (heavy shock)	2.0 - 3.0	1.5 - 3.0

ASME code also suggests about the allowable design stress, $\eta_{allowable}$ to be considered for steel shafting,

ASME Code for commercial steel shafting

= 55 MPa for shaft without keyway

= 40 MPa for shaft with keyway

ASME Code for steel purchased under definite specifications

= 30% of the yield strength but not over 18% of the ultimate strength in tension for shafts without keyways. These values are to be reduced by 25% for the presence of keyways.

Design of Shafts on the basis of Rigidity

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. **Torsional rigidity.** The torsional rigidity is important in the case of camshaft of an I.C.engine where the timing of the valves would be affected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting

value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft. The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G.\theta}{L} \quad \text{or} \quad \theta = \frac{T.L}{J.G}$$

Where, θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. Lateral rigidity. It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *i.e.*

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

BIS codes of Shafts

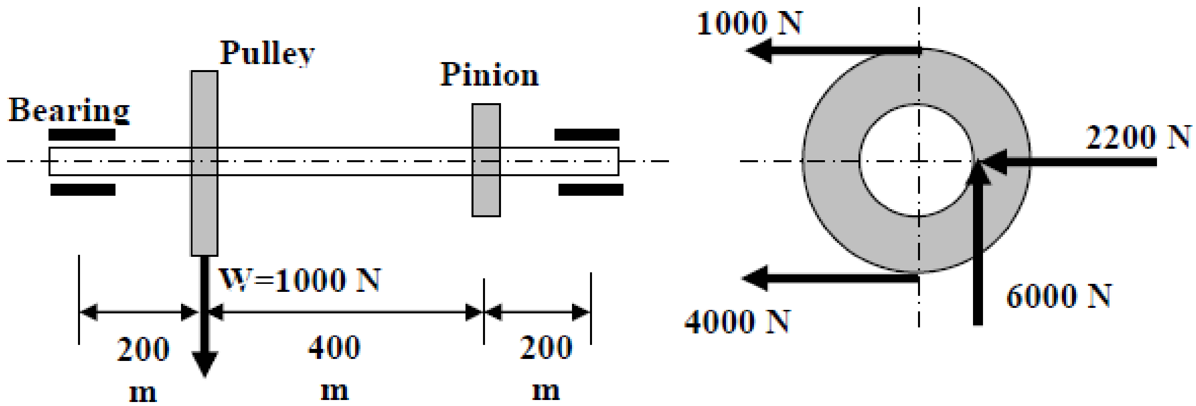
The standard sizes of transmission shafts are:

25mm to 60mm with 5mm steps; 60mm to 110mm with 10mm steps ; 110mm to 140mm with 15mm steps ; and 140mm to 500mm with 20mm steps.

The standard length of the shafts are 5m, 6m and 7m.

Problems:

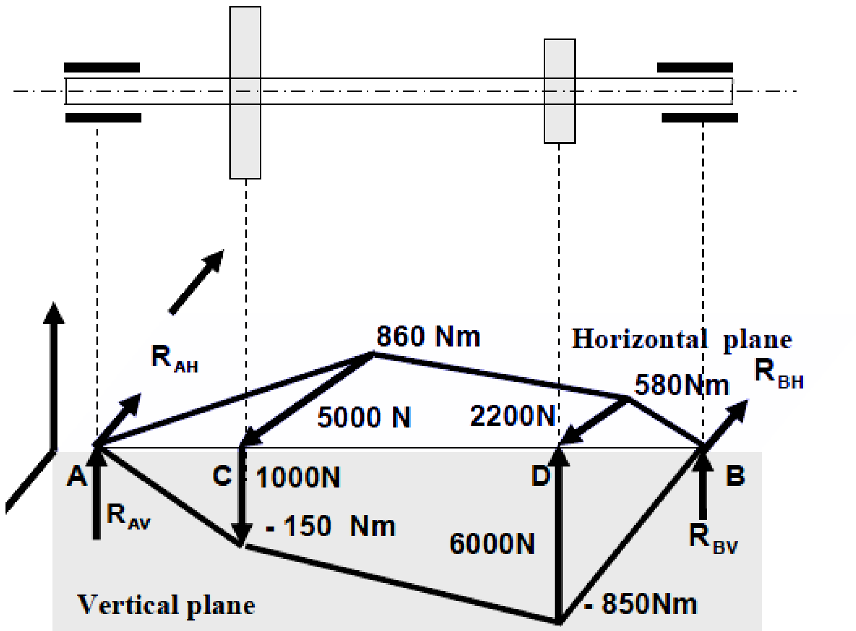
Q. The problem is shown in the given figure. A pulley drive is transmitting power to a pinion, which in turn is transmitting power to some other machine element. Pulley and pinion diameters are 400mm and 200mm respectively. Shaft has to be designed for minor to heavy shock.



From the given figure, the magnitude of torque,

$$T = (4000 - 1000) \times 200 \text{ N mm} = 600 \times 10^3 \text{ Nmm}$$

It is observed that the load on the shaft is acting both in horizontal and vertical planes. The loading diagram, corresponding bearing reactions and bending moment diagram is given below.



Loading and Bending Moment Diagram

The bending moment at C:

For vertical plane, M_V: -150 Nm

For horizontal plane, M_H: 860 Nm

Resultant moment: 873 Nm

The bending moment at D:

For vertical plane, M_V: -850 Nm

For horizontal plane, M_H: 580 Nm

Resultant moment: 1029Nm

Therefore, section-D is critical and where bending moment and torsion is 1029 Nm and 600 Nm respectively.

ASME code for shaft design is suitable in this case as no other specifications are provided. In absence of any data for material property, the allowable shear for commercial steel shaft may be taken as 40 MPa, where keyway is present in the shaft.

For the given condition of shock, let us consider C_{bm} = 2.0 and C_t = 1.5.

From the ASME design code, we have,

$$d_o^{\,3} = \frac{16\times10^3}{\tau_d\times\pi}\left(\sqrt{(C_{bm}\times1029)^2+(C_t\times600)^2}\right)$$

$$= \frac{16\times10^3}{40\times\pi}\left(\sqrt{(2.0\times1029)^2+(1.5\times600)^2}\right)$$

$$\therefore d_o = 65.88\text{ mm} \approx 66\text{mm}$$

From standard size available, the value of shaft diameter is also 66mm.

Q. A mild steel shaft transmits 20 kW at 200 rpm. It is subjected to a bending moment of 562.5 N-m. Determine the size of the shaft, if the allowable shear stress is 42 MPa, and the maximum tensile or compressive stress is not to exceed 58 MPa. What size of the shaft will be required if it is subjected to gradually applied load.

Solution: Given,

$$Power\text{ "}P = 20\,kW\text{"}, Rotational\ speed\ N = 200\ rpm;\quad Bending\ moment\ M = 562.5\ N\text{-}m;$$

$$Alloable\ shear\ stress\ \tau_{all} = 42MPa;\quad Normal\ stress\ \sigma_{all} = 58\ MPa$$

Torque :

$$T = \frac{kW\times10^6\times60}{2\pi N} = \frac{20\times10^6\times60}{2\times\pi\times200} = 954929.6585\ N\text{-}mm;$$

Equivalent Bending Moment :

$$M_e = \frac{1}{2}\left[M + \sqrt{\left(M^2 + T^2\right)}\right] = \frac{1}{2}\left[562500 + \sqrt{\left(562500^2 + 954929.6585^2\right)}\right] =$$

$$M_e = \frac{1}{2}[562500 + 1108285.569] = 835392.7845 \ \ N\text{ - }mm$$

Equivalent Twisting Moment :

$$T_e = \sqrt{\left(M^2 + T^2\right)} = \sqrt{\left(562500^2 + 954929.6585^2\right)} = 1108285.569\ N\text{ - }mm;$$

The maximum shear stress induced in the material of the shaft

$$\tau_{max} = \frac{16T_e}{\pi d^3} = \frac{16 \times 1108285.569}{\pi d^3} = \frac{5644452.053}{d^3}$$

Hence using the maximum shear stress theory

$$42 \geq \frac{5644452.053}{d^3} \Rightarrow d^3 \geq \frac{5644452.053}{42}$$

$$d^3 \geq 134391.7156$$

$$d \geq 51.22\ mm;$$

The maximum normal (tensile or compressive) stress induced in the material of the shaft

$$\sigma_{max} = \frac{32M_e}{\pi d^3} = \frac{32 \times 835392.7845}{\pi d^3} = \frac{8509241.029}{d^3}$$

Hence using the maximum normal stress theory

$$58 \geq \frac{8509241.029}{d^3} \Rightarrow d^3 \geq \frac{8509241.029}{58}$$

$$d^3 \geq 146711.0522$$

$$d \geq 52.74\ mm;$$

Considering the gradual application of the load :

Assu min g :

shock and fatigue factor for bending and torsion $K_t = 1.0$

Then shock and fatigue factor for bending $K_b = 1.5$

Equivalent Bending Moment :

$$M_e = \frac{1}{2}\left[K_b M + \sqrt{\left((K_b M)^2 + (K_t T)^2\right)}\right] = \frac{1}{2}\left[1.5 \times 562500 + \sqrt{\left(1.5 \times 562500^2 + (1 \times 954929.6585\right.}\right.$$

$$M_e = 1059016.483\ \ N\text{ - }mm$$

Equivalent Twisting Moment :

$$T_e = \sqrt{\left((K_b M)^2 + (K_t T)^2\right)} = \sqrt{\left(1.5 \times 562500^2 + (1 \times 954929.6585)^2\right)}$$

$$T_e = 1274285.963\ N\text{ - }mm$$

The maximum shear stress induced in the material of the shaft

$$\tau_{max} = \frac{16T_e}{\pi d^3} = \frac{16 \times 1274285.963}{\pi d^3} = \frac{6489885.117}{d^3}$$

Hence using the maximum shear stress theory

$$42 \geq \frac{6489885.117}{d^3} \Rightarrow d^3 \geq \frac{6489885.117}{42}$$

$$d^3 \geq 154521.0742$$

$$d \geq 53.66 \text{ mm;}$$

The maximum normal (tensile or compressive) stress induced in the material of the shaft

$$\sigma_{max} = \frac{32M_e}{\pi d^3} = \frac{32 \times 1059016.483}{\pi d^3} = \frac{10787053.32}{d^3}$$

Hence using the maximum normal stress theory

$$58 \geq \frac{10787053.32}{d^3} \Rightarrow d^3 \geq \frac{10787053.32}{58}$$

$$d^3 \geq 185983.6779$$

$$d \geq 57.08 \text{ mm;}$$

Hence the diameter of the shaft is 57.08 mm or 58 mm.

Q. A shaft is supported by two bearings placed at a distance of 600 mm diameter pulley is mounted at a distance of 300mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having diameter is placed 200 mm to the left of right hand bearing anf is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and μ = 0.24. Determine suitable diameter of solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.

Solution. Given : $AB = 800$ mm ; $\alpha_C = 20°$; $D_C = 600$ mm or $R_C = 300$ mm ; $AC = 200$ mm ; $D_D = 700$ mm or $R_D = 350$ mm ; $DB = 250$ mm ; $\theta = 180° = \pi$ rad ; $W = 2000$ N ; $T_1 = 3000$ N ; $T_1/T_2 = 3$; $\tau = 40$ MPa $= 40$ N/mm²

The space diagram of the shaft is shown in Fig (*a*).

We know that the torque acting on the shaft at *D*,

$$T = (T_1 - T_2)\, R_D = T_1\left(1 - \frac{T_2}{T_1}\right) R_D$$

$$= 3000\left(1 - \frac{1}{3}\right) 350 = 700 \times 10^3 \text{ N-mm} \qquad \ldots(\because T_1/T_2 = 3)$$

The torque diagram is shown in Fig. (*b*).

Assuming that the torque at *D* is equal to the torque at *C*, therefore the tangential force acting on the gear *C*,

$$F_{tc} = \frac{T}{R_C} = \frac{700 \times 10^3}{300} = 2333 \text{ N}$$

and the normal load acting on the tooth of gear C,

$$W_C = \frac{F_{tc}}{\cos \alpha_C} = \frac{2333}{\cos 20°} = \frac{2333}{0.9397} = 2483 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. Resolving the normal load vertically and horizontally, we get

Vertical component of W_C *i.e.* the vertical load acting on the shaft at C,

$$W_{CV} = W_C \cos 20°$$
$$= 2483 \times 0.9397 = 2333 \text{ N}$$

and horizontal component of W_C *i.e.* the horizontal load acting on the shaft at C,

$$W_{CH} = W_C \sin 20°$$
$$= 2483 \times 0.342 = 849 \text{ N}$$

Since $T_1 / T_2 = 3$ and $T_1 = 3000$ N, therefore

$$T_2 = T_1 / 3 = 3000 / 3 = 1000 \text{ N}$$

∴ Horizontal load acting on the shaft at D,

$$W_{DH} = T_1 + T_2 = 3000 + 1000 = 4000 \text{ N}$$

and vertical load acting on the shaft at D,

$$W_{DV} = W = 2000 \text{ N}$$

The vertical and horizontal load diagram at C and D is shown in Fig. 14.6 (c) and (d) respectively.

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all considering the vertical loading at C and D. Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that

$$R_{AV} + R_{BV} = 2333 + 2000 = 4333 \text{ N}$$

Taking moments about A, we get

$$R_{BV} \times 800 - 2000\,(800 - 250) + 2333 \times 200$$
$$= 1\,566\,600$$

$$\therefore \quad R_{BV} = 1\,566\,600 / 800 = 1958 \text{ N}$$

and $\quad R_{AV} = 4333 - 1958 = 2375 \text{ N}$

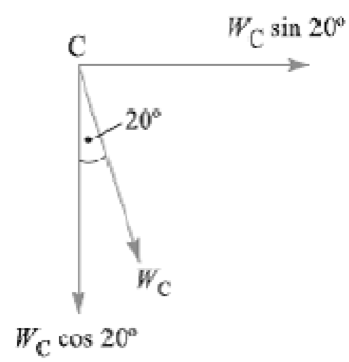


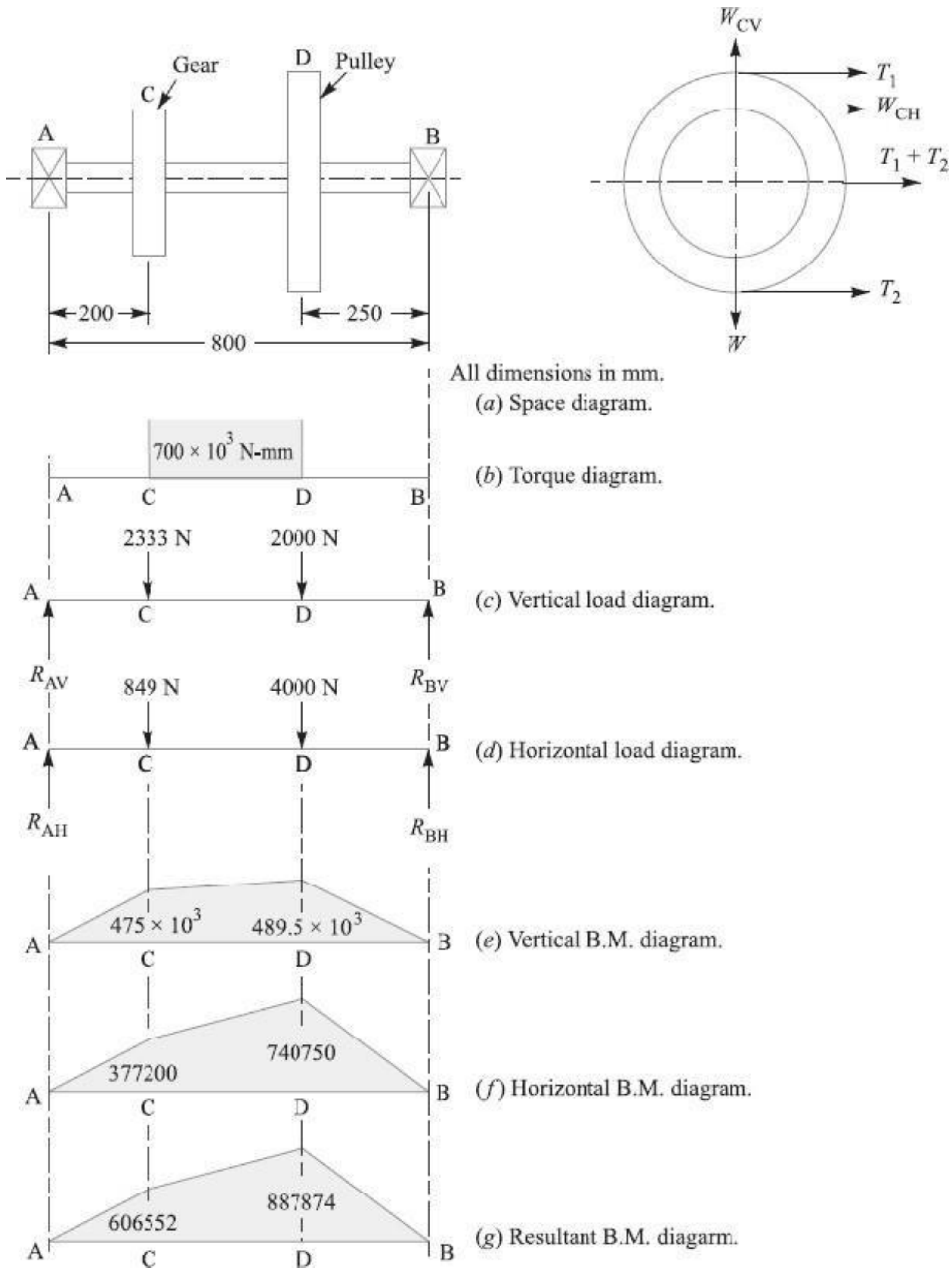
Fig. 14.7

We know that B.M. at A and B,

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $\quad M_{CV} = R_{AV} \times 200 = 2375 \times 200$
$$= 475 \times 10^3 \text{ N-mm}$$

B.M. at D, $\quad M_{DV} = R_{BV} \times 250 = 1958 \times 250 = 489.5 \times 10^3$ N-mm



The bending moment diagram for vertical loading is shown in Fig. 14.6 (*e*).

Now consider the horizontal loading at *C* and *D*. Let R_{AH} and R_{BH} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AH} + R_{BH} = 849 + 4000 = 4849 \text{ N}$$

Taking moments about A, we get

$$R_{BH} \times 800 = 4000\,(800 - 250) + 849 \times 200 = 2\,369\,800$$

$\therefore$ $R_{BH} = 2\,369\,800 / 800 = 2963$ N

and $R_{AH} = 4849 - 2963 = 1886$ N

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $M_{CH} = R_{AH} \times 200 = 1886 \times 200 = 377\,200$ N-mm

B.M. at D, $M_{DH} = R_{BH} \times 250 = 2963 \times 250 = 740\,750$ N-mm

The bending moment diagram for horizontal loading is shown in Fig. 14.6 (f).

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(175 \times 10^3)^2 + (377\,200)^2}$$

$$= 606\,552 \text{ N-mm}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(489.5 \times 10^3)^2 + (740\,750)^2}$$

$$= 887\,874 \text{ N-mm}$$

Maximum bending moment

The resultant B.M. diagram is shown in Fig. 14.6 (g). We see that the bending moment is maximum at D, therefore

Maximum B.M., $M = M_D = 887\,874$ N-mm **Ans.**

Diameter of the shaft

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(887\,874)^2 + (700 \times 10^3)^2} = 1131 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1131 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 1131 \times 10^3 / 7.86 = 144 \times 10^3$ or $d = 52.4$ say 55 mm **Ans.**

Problem:

A hollow shaft is subjected to a maximum torque of 1.5 kN-m and a maximum bending moment of 3 kN-m. It is subjected, at the same time, to an axial load of 10 kN. Assume that the load is applied gradually and the ratio of the inner diameter to the outer diameter is 0.5. If the outer diameter of the shaft is 80 mm, find the shear stress induced in the shaft.

Solution. Given: $T = 1.5$ kN-m $= 1.5 \times 10^3$ N-m ; $M = 3$ kN-m $= 3 \times 10^3$ N-m ;

$F = 10$ kN $= 10 \times 10^3$ N ; $k = d_i / d_o = 0.5$; $d_o = 80$ mm $= 0.08$ m

Let τ = Shear stress induced in the shaft.

Since the load is applied gradually, therefore from DDB, we find that K_m =1.5 ; and K_t = 1.0

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o (1+k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 3 \times 10^3 + \frac{1 \times 10 \times 10^3 \times 0.08 (1+0.5^2)}{8}\right]^2 + (1 \times 1.5 \times 10^3)^2}$$

$$= \sqrt{(4500 + 125)^2 + (1500)^2} = 4862 \text{ N-m} = 4862 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$4862 \times 10^3 = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4) = \frac{\pi}{16} \times \tau (80)^3 (1 - 0.5^4) = 94\,260\,\tau$$

$$\therefore \qquad \tau = 4862 \times 10^3 / 94\,260 = 51.6 \text{ N/mm}^2 = 51.6 \text{ MPa } \textbf{Ans.}$$

Problem:

A hollow shaft of 0.5 m outside diameter and 0.3 m inside diameter is used to drive a propeller of a marine vessel. The shaft is mounted on bearings 6 metre apart and it transmits 5600 kW at 150 r.p.m. The maximum axial propeller thrust is 500 kN and the shaft weighs 70 kN.

Determine:

1. The maximum shear stress developed in the shaft, and

2. The angular twist between the bearings.

Solution. Given : d_o = 0.5 m ; d_i = 0.3 m ; P = 5600 kW = 5600 × 10^3 W ; L = 6 m ; N = 150 r.p.m. ; F = 500 kN = 500 × 10^3 N ; W = 70 kN = 70 × 10^3 N

1. *Maximum shear stress developed in the shaft*

Let τ = Maximum shear stress developed in the shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{5600 \times 10^3 \times 60}{2\pi \times 150} = 356\,460 \text{ N-m}$$

and the maximum bending moment,

$$M = \frac{W \times L}{8} = \frac{70 \times 10^3 \times 6}{8} = 52\,500 \text{ N-m}$$

Now let us find out the column factor α. We know that least radius of gyration,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]}}$$

$$= \frac{1}{4}\sqrt{(d_o)^2 + (d_i)^2} = \frac{1}{4}\sqrt{(0.5)^2 + (0.3)^2} = 0.1458 \text{ m}$$

∴ Slenderness ratio,

$$L / K = 6 / 0.1458 = 41.15$$

and column factor,

$$\alpha = \frac{1}{1 - 0.0044\left(\frac{L}{K}\right)} \qquad \ldots\left(\because \frac{L}{K} < 115\right)$$

$$= \frac{1}{1 - 0.0044 \times 41.15} = \frac{1}{1 - 0.18} = 1.22$$

Assuming that the load is applied gradually, therefore from Table 14.2, we find that

$$K_m = 1.5 \text{ and } K_t = 1.0$$

Also $\quad k = d_i / d_o = 0.3 / 0.5 = 0.6$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o (1 + k^2)}{8}\right]^2 \quad (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 52\ 500 + \frac{1.22 \times 500 \times 10^3 \times 0.5\ (1 + 0.6^2)}{8}\right]^2 + (1 \times 356\ 460)^2}$$

$$= \sqrt{(78\ 750 + 51\ 850)^2 + (356\ 460)^2} = 380 \times 10^3 \text{ N-m}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$380 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) = \frac{\pi}{16} \times \tau\,(0.5)^3\,[1 - (0.6)^4] = 0.02\,\tau$$

∴ $\quad \tau = 380 \times 10^3 / 0.02 = 19 \times 10^6 \text{ N/m}^2 = 19$ MPa **Ans.**

2. *Angular twist between the bearings*

Let $\quad \theta$ = Angular twist between the bearings in radians.

We know that the polar moment of inertia for a hollow shaft,

$$J = \frac{\pi}{32}[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}[(0.5)^4 - (0.3)^4] = 0.005\ 34 \text{ m}^4$$

From the torsion equation,

$$\frac{T}{J} = \frac{G \times \theta}{L}, \text{ we have}$$

$$\theta = \frac{T \times L}{G \times J} = \frac{356\ 460 \times 6}{84 \times 10^9 \times 0.00\ 534} = 0.0048 \text{ rad}$$

... (Taking $G = 84$ GPa $= 84 \times 10^9$ N/m^2)

$$= 0.0048 \times \frac{180}{\pi} = 0.275° \textbf{ Ans.}$$

Problem:

A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.

Solution. Given : $P = 4$ kW = 4000 W ; $N = 800$ r.p.m. ; $\theta = 0.25° = 0.25 \times \frac{\pi}{180} = 0.0044$ rad ; $L = 1$ m = 1000 mm ; $G = 84$ GPa $= 84 \times 10^9$ N/m^2 $= 84 \times 10^3$ N/mm^2

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\,740 \text{ N-mm}$$

We also know that $\frac{T}{J} = \frac{G \times \theta}{L}$ or $J = \frac{T \times l}{G \times \theta}$

or $$\frac{\pi}{32} \times d^4 = \frac{47\,740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\,167$$

$\therefore$ $d^4 = 129\,167 \times 32 / \pi = 1.3 \times 10^6$ or $d = 33.87$ say 35 mm **Ans.**

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\,740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\,(35)^3 = 8420\,\tau$$

$\therefore$ $\tau = 47\,740 / 8420 = 5.67$ N/mm^2 = 5.67 MPa **Ans.**

Problem:

Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.

Solution. Given : $d_o = d$; $d_i = d_o / 2$ or $k = d_i / d_o = 1/2 = 0.5$

Comparison of weight

We know that weight of a hollow shaft,

$$W_H = \text{Cross-sectional area} \times \text{Length} \times \text{Density}$$

$$= \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] \times \text{Length} \times \text{Density} \qquad ...(i)$$

and weight of the solid shaft,

$$W_S = \frac{\pi}{4} \times d^2 \times \text{Length} \times \text{Density} \qquad ...(ii)$$

Since both the shafts have the same material and length, therefore by dividing equation (*i*) by equation (*ii*), we get

$$\frac{W_H}{W_S} = \frac{(d_o)^2 - (d_i)^2}{d^2} = \frac{(d_o)^2 - (d_i)^2}{(d_o)^2} \qquad ...(\because d = d_o)$$

$$= 1 - \frac{(d_i)^2}{(d_o)_2} = 1 - k^2 = 1 - (0.5)^2 = 0.75 \textbf{ Ans.}$$

Comparison of Strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4) \qquad ...(iii)$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(iv)$$

Dividing equation (*iii*) by equation (*iv*), we get

$$\frac{T_H}{T_S} = \frac{(d_o)^3 (1 - k^4)}{d^3} = \frac{(d_o)^3 (1 - k^4)}{(d_o)^3} = 1 - k^4 \qquad ...(\because d = d_o)$$

$$= 1 - (0.5)^4 = 0.9375 \text{ Ans.}$$

Comparison of stiffness

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

$\therefore$ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32} \left[(d_o)^4 - (d_i)^4 \right] \qquad ...(v)$$

and stiffness of a solid shaft,

$$S_S = \frac{G}{L} \times \frac{\pi}{32} \times d^4 \qquad ...(vi)$$

Dividing equation (*v*) by equation (*vi*), we get

$$\frac{S_H}{S_S} = \frac{(d_o)^4 - (d_i)^4}{d^4} = \frac{(d_o)^4 - (d_i)^4}{(d_o)^4} = 1 - \frac{(d_i)^4}{(d_o)^4} \qquad ...(\because d = d_o)$$

$$= 1 - k^4 = 1 - (0.5)^4 = 0.9375 \text{ Ans.}$$

Problem:

Prove that a hollow shaft is stronger and stiffer than a solid shaft of same length, weight and material.

Solution: (i) Strength

$$\text{Wt of the shaft } W = \frac{\pi}{4} D^2 lr^2 \times S$$

Wt of solid shaft W_s and weight of hollow shaft W_H

$$\text{i.e., } D^2 = \left(D_o^2 - D_i^2\right) \text{or}$$

$$D^3 = D\left(D_o^2 - D_i^2\right)$$

Torque transmitted by solid shaft

$$M_{t_s} = \frac{\pi}{16} D^3 \tau_d$$

Torque transmitted by hollow shaft

$$M_{t_s} = \frac{\pi}{16}\left(D_o^4 - D_i^4\right)\tau_d$$

$$\text{Now ratio of tarque} = \frac{M_{th}}{M_{ts}} = \frac{D_o^4 - D_i^4}{D_0} \times \frac{1}{D\left(D_o^2 - D_i^2\right)}$$

$$= \frac{D_o^2 - D_i^2}{DD_o} = \frac{D_o^2 - D_i^2}{D_o\sqrt{D_o^2 - D_i^2}} = \frac{D_o^2\left(1+k^2\right)}{D_o \times D_o\sqrt{1-k^2}}$$

$$\text{where } k = \frac{D_i}{D_o}$$

$$\frac{M_{tH}}{M_{ts}} = \frac{1+k^2}{\sqrt{1-k^2}} > 1, \text{Hence hollow shaft is stronger than the solid shown of same weight}$$

$$\text{ii) Stiffness: Tarsional stiffness of solid shaft } K_s = \frac{G}{L} \times \frac{\pi}{32} D^4$$

$$\text{Tarsional stiffness of hollow shaft } K_H = \frac{G}{L} \times \frac{\pi}{32}\left(D_0^4 - D_i^4\right)$$

$$\text{Ratio of stiffness } \frac{k_H}{k_s} = \frac{D_0^4 - D_i^4}{D^4} = \frac{D_0^2\left(1+k^2\right)}{D_0^2\left(1-k^2\right)} = \frac{1+k}{1-k} > 1$$

Hence hollow shaft is stiffer than a solid shaft of same length.

Problem:

Commercial shaft 1 metre long supported between bearings has a pulley of 600 mm diameter weighing 1 kN, driven by a horizontal belt drive keyed to the shaft at a distance of 400 mm to the left of the right bearing and receives 25 kW at 1000rpm. Power from the shaft is transmitted from the 20·, spur pinion of a pitch circle diameter 200 mm which is mounted at 200 mm to the right of the left bearing to a gear such that tangential force on the gear acts vertically upwards. Take the ratio of the belt tension is 3. Determine the standard size of the shaft based on maximum shear stress theory assume Cm= 1.75, CT = 1.25.

Solution:

Shaft is subjected to combined Bending & Torsion. Bending is due to weight of pulley, belt tension & gear forces, where as torsion due to power transmitted)

$$\left\{\begin{array}{l}\text{E 14.12 Diameter of Shaft} \\ \\ \text{according to Max. Shear. Stress}\end{array}\right\} D = \left[\frac{16}{\pi\tau_{ed}}\left\{\left(K_b M_b\right)^2 + \left(K_t M_t\right)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

$\tau_{ed} = 40MPa$;by assuming C10 (Table 1.5)

$$\sigma_Y = 205;\ \sigma_{YD} = \frac{205}{2.5} = 82;\ \tau_{ed} = \frac{\sigma_{Yd}}{2} = 41 \simeq 40MPa$$

$C_m = K_b = 1.75$; T 14.2 assuming minor shock for rotating shaft ; $K_b = 1.5$ to $2 = 1.75$ and
$K_t = 1.0$ to $1.5 = 1.25$

$C_t = K_t = 1.25$

$$M_t = \frac{9550N \times 10^3}{1000} = 9550 \times 25 \times \frac{1000}{1000} = 238750Nmm$$

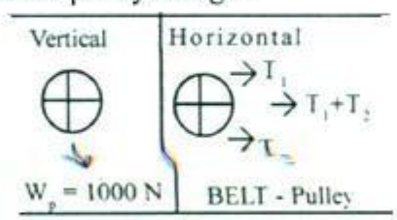
To Find M_b

We have to Determine all vertical & horizontal forces of both pulley and gear

(i) At Belt Pulley

(a) Vertical force $\Rightarrow W_p$ = weight of pulley = 1000 N ↓

(b) Horizontal force =T1 +T2 = 1591.68N --->



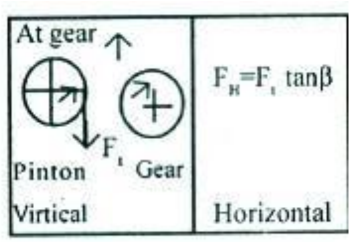
$$Use\ \frac{T_1}{T_2} = e^{M\theta},\ M_t = (T_1 - T_2)r \Rightarrow 238750 = (3T_2 - T_2) \times \left[\frac{600}{2}\right] \Rightarrow T_2 = 397.92N$$

& T_1=1193.76 N

Total $\mathbf{T_1 + T_2 = 1591.68\ N}$

(ii) At Pinion

(a) Vertical force



$$[\Rightarrow F_t = \frac{M_t}{r_g} = \frac{238750}{\left(\frac{200}{2}\right)} = 2387.5N \downarrow$$

(b) Horizontal force $\Rightarrow F_H = F_t \tan\beta = 2387.5 \tan 20^0 = 868.99\ N$

($\beta = 20^0$ Pressure angle)

Horizontal load diagram

fig (a)

Horizontal force at Pinion	=	1591.68 N
Horizontal force at Pulley	=	868.99 N
Total	=	**2460.67 N**

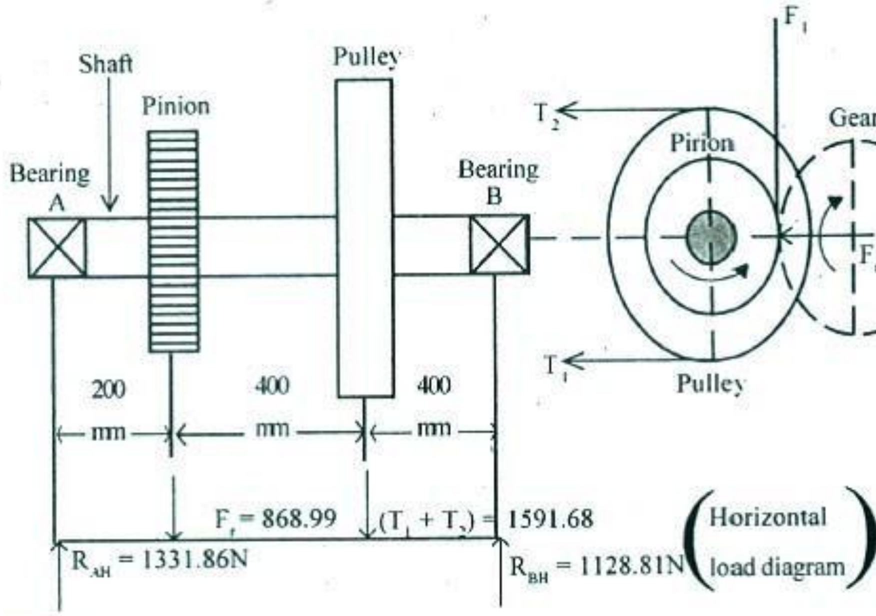


Fig (a)

Fig (b)

Bending moment Diagram

$R_{BH} \times 1000 = 869.99 \times 200 + 1591.68 \times 600 \Rightarrow R_{BH} = 1128.81$ N

Taking moments at A

R_{AH} = Tot. load - R_{BH} = 2460.67-1128.81 $\Rightarrow R_{AH}$ = 1331.86 N

BM at A = 0 = 0

BM at Pinion = $R_{AH} \times 200 = 1331.86 \times 200$ = 266372 N mm

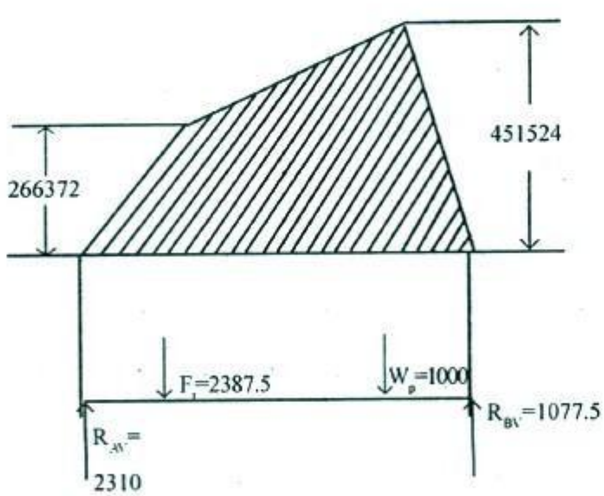
BM at Pulley = $R_{BH} \times 400 = 1128.81 \times 400$ = 451524 N mm.

Fig (c)

Vertical load diagram Vertical force at pinion $\Rightarrow F_t$ = 2387.5 N

Vertical force at Pulley $\Rightarrow W_p$ = 1000.0 N

Total = 3387.5 N



Vertical

Load Diagram (c)

Fig (d)

Bending Moment Diagram

Taking moment at A $R_{BV} \times 1000 = 1000 \times 600 + 2387 \times 200 \Rightarrow R_{BV} = \mathbf{1077.5\ N}$

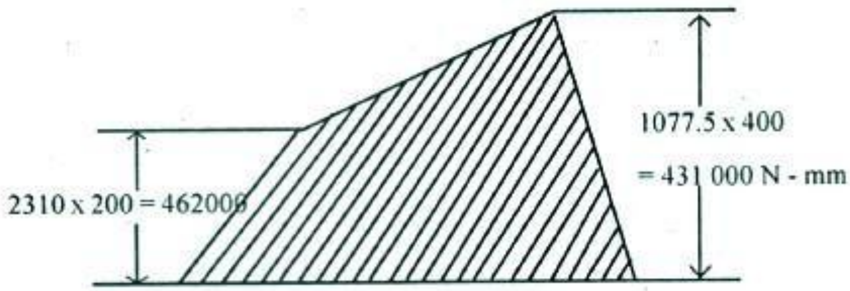
$R_{AV} = \text{Tot. load} - R_{BV} = 3387.5 - 1077.5 \Rightarrow R_{AV} = \mathbf{2310\ N}$

BM at A = 0

BM at pinion = $R_{AV} \times 200 = 2310 \times 200 = 462000$ N mm

BM at pulley = $R_{BV} \times 400 = 1077.5 \times 400 = 431000$ N mm

BM at B = 0



Vertical Bending Moment Diagram Fig (d)

Resultant BM

Fig (e) (i) Resultant at Pinion = $\sqrt{266372^2 + 462000^2} = 533289.98\ Nmm$ (min)

(ii) Resultant at Pulley = $\sqrt{451529^2 + 431000^2} = 624207.44\ Nmm$ (max)

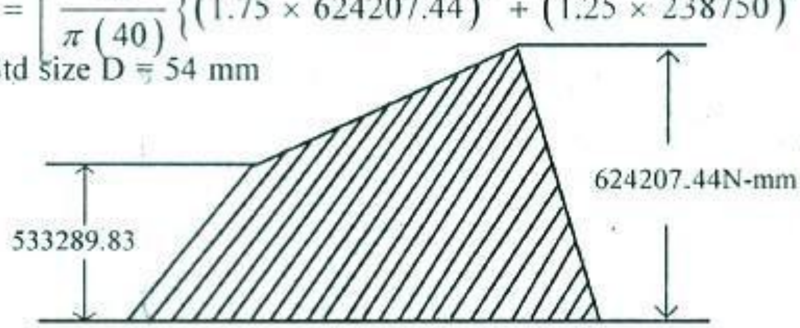
Design to be based on max Resultant $M_B = 624207.44$ N mm

Now apply into eqn

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(K_b M_b \right)^2 + \left(K_t M_t \right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

$$= \left[\frac{16}{\pi (40)} \left\{ \left(1.75 \times 624207.44 \right)^2 + \left(1.25 \times 238750 \right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} = 52.44\ mm$$

Use Std size D = 54 mm



Resultant bending moment diagram Fig (e)

UNIT 6

COTTER AND KNUCKLE JOINTS, KEYS AND COUPLINGS

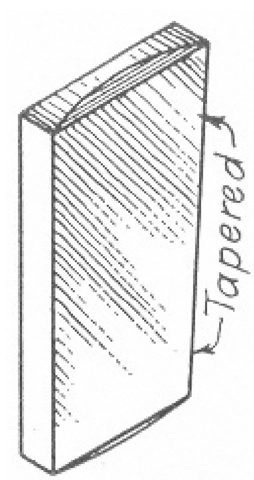
Instructional Objectives

- *A typical cotter joint, its components and working principle.*
- *Detailed design procedure of a cotter joint.*
- *A typical knuckle joint, its components and working principle.*
- *Detailed design procedure of a knuckle joint.*
- *Different types of keys and their applications.*
- *Detailed design procedure of a typical rigid flange coupling.*
- *Detailed design procedure of a typical flexible rubber-bush coupling.*

Cotter joint

A cotter is a flat wedge-shaped piece of steel as shown in figure. This is used to connect rigidly two rods which transmit motion in the axial direction, without rotation. These joints may be subjected to tensile or compressive forces along the axes of the rods.

Examples of cotter joint connections are: connection of piston rod to the crosshead of a steam engine, valve rod and its stem etc.



A typical cotter joint is as shown in figure. One of the rods has a socket end into which the other rod is inserted and the cotter is driven into a slot, made in both the socket and the rod.

The cotter tapers in width (usually 1:24) on one side only and when this is driven in, the rod is forced into the socket. However, if the taper is provided on both the edges it must be less than the sum of the friction angles for both the edges to make itself locking i.e., $\alpha 1 + \alpha 2 < \theta 1 + \theta 2$ where $\alpha 1$, $\alpha 2$ are the angles of taper on the rod edge and socket edge of the cotter respectively and $\theta 1$, $\theta 2$ are the corresponding angles of friction. This also means that if taper is given on one side only then $\alpha < \theta_1 + \theta_2$ for self locking. Clearances between the cotter and slots in the rod end and socket allows the driven cotter to draw together the two parts of the joint until the socket end comes in contact with the cotter on the rod end.

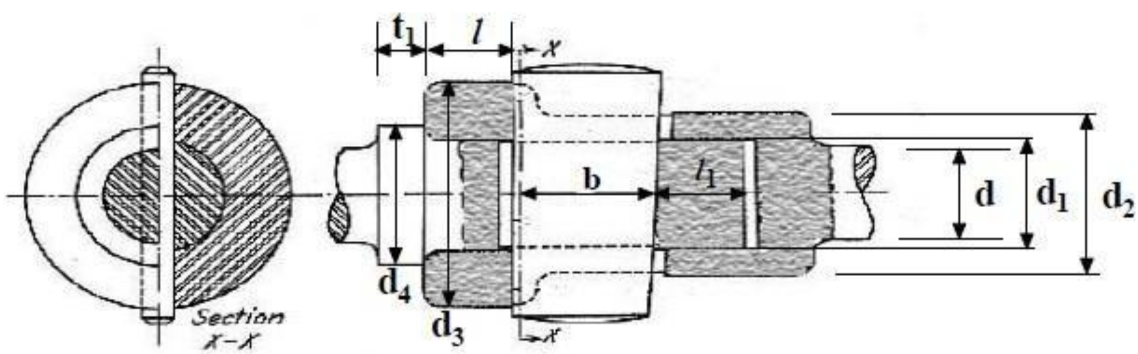


Fig: Cross-sectional views of a typical cotter joint

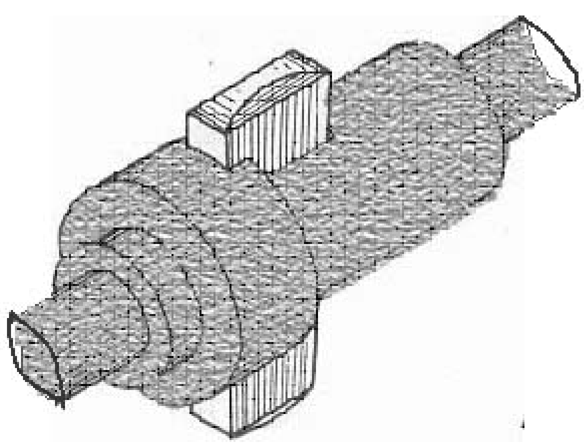


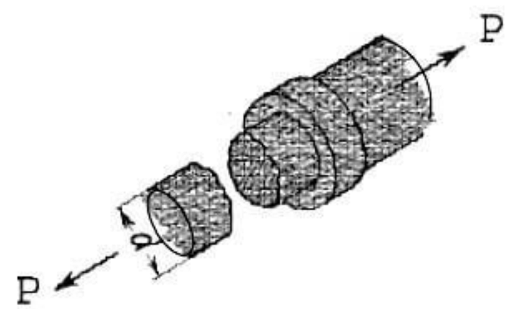
Fig: An isometric view of a typical cotter joint

Design of a cotter joint

If the allowable stresses in tension, compression and shear for the socket, rod and cotter be ζ_t, ζ_c and η respectively, assuming that they are all made of the same material, we may write the following failure criteria:

1. Tension failure of rod at diameter d

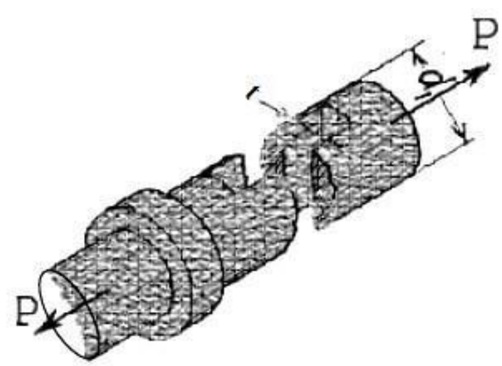
$$\frac{\pi}{4} d^2 \sigma_t = P$$



Tension failure of the rod

2. Tension failure of rod across slot

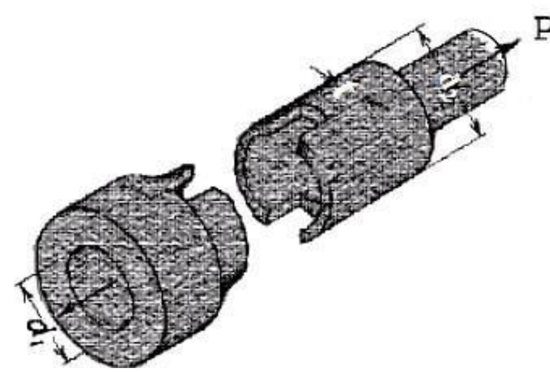
$$\left(\frac{\pi}{4} d_1^2 - d_1 t\right) \sigma_t = P$$



Tension failure of rod across slot

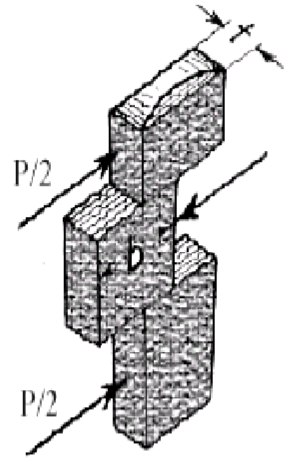
3. Tensile failure of socket across slot

$$\left(\frac{\pi}{4}(d_2^2 - d_1^2) - (d_2 - d_1)t\right) \sigma_t = P$$



4. Shear failure of cotter

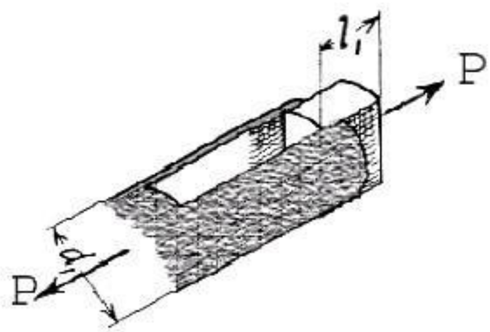
$$2bt\tau = P$$



Shear failure of cotter

5. Shear failure of rod end

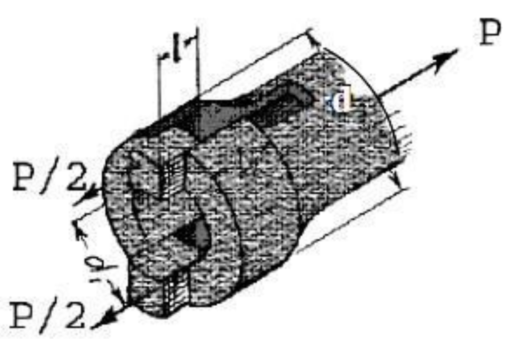
$$2l_1 d_1 \tau = P$$



Shear failure of rod end

6. Shear failure of socket end

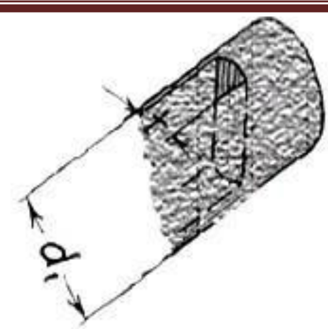
$$2l\left(d_3 - d_1\right)\tau = P$$



Shear failure of socket end

7. Crushing failure of rod or cotter

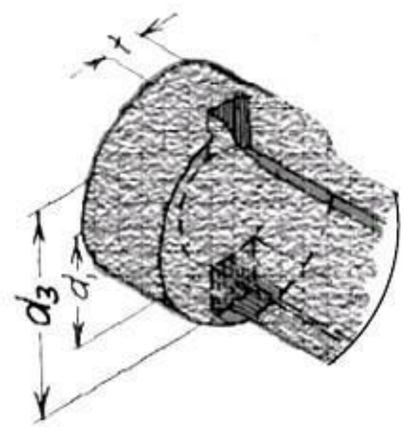
$d_1 t \sigma_c = P$



Crushing failure of rod or cotter

8. Crushing failure of socket or rod

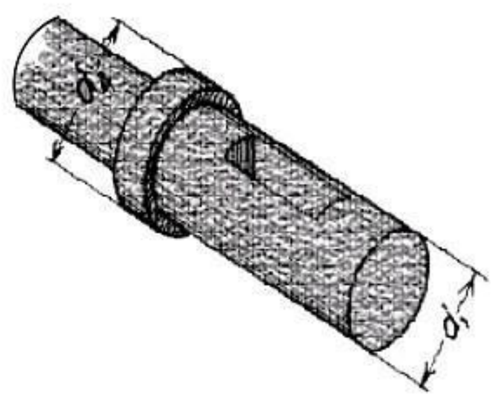
$(d_3 - d_1) t \sigma_c = P$



Crushing failure of socket or rod

9. Crushing failure of collar

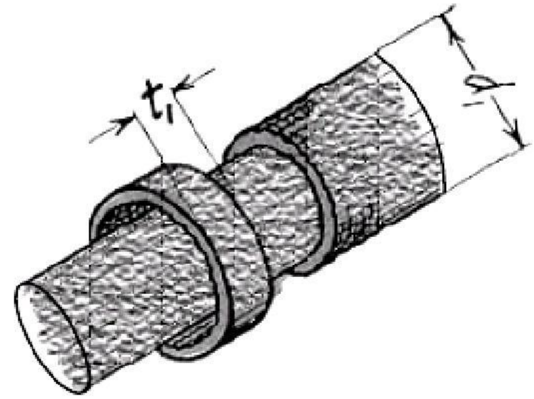
$\left(\frac{\pi}{4}(d_4^2 - d_1^2)\right)\sigma_c = P$



Crushing failure of collar

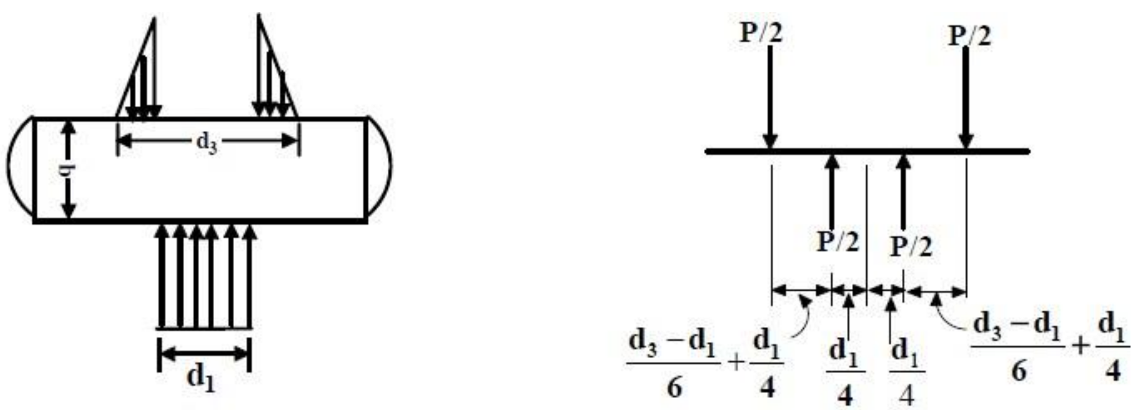
10. Shear failure of collar

$$\pi d_1 t_1 \tau = P$$



Shear failure of collar

Cotters may bend when driven into position. When this occurs, the bending moment cannot be correctly estimated since the pressure distribution is not known. However, if we assume a triangular pressure distribution over the rod, as shown in figure.



(a) (b)

This gives maximum bending moment = $\frac{P}{2}\left(\frac{d_3 - d_1}{6} + \frac{d_1}{4}\right)$ and

The bending stress, $\sigma_b = \dfrac{\frac{P}{2}\left(\frac{d_3 - d_1}{6} + \frac{d_1}{4}\right)\frac{b}{2}}{\frac{tb^3}{12}} = \dfrac{3P\left(\frac{d_3 - d_1}{6} + \frac{d_1}{4}\right)}{tb^2}$

Tightening of cotter introduces initial stresses which are again difficult to estimate. Sometimes therefore it is necessary to use empirical proportions to design the joint. Some typical proportions are given below:

$$d_1 = 1.21.d$$

$$d_2 = 1.75.d$$

$$d_3 = 2.4\ d$$

$$d_4 = 1.5.d$$

$$t = 0.31d$$

$$b = 1.6d$$

$$l = l_1 = 0.75d$$

$$t_1 = 0.45d$$

s= clearance

Knuckle Joint

A knuckle joint is used to connect two rods under tensile load. This joint permits angular misalignment of the rods and may take compressive load if it is guided.

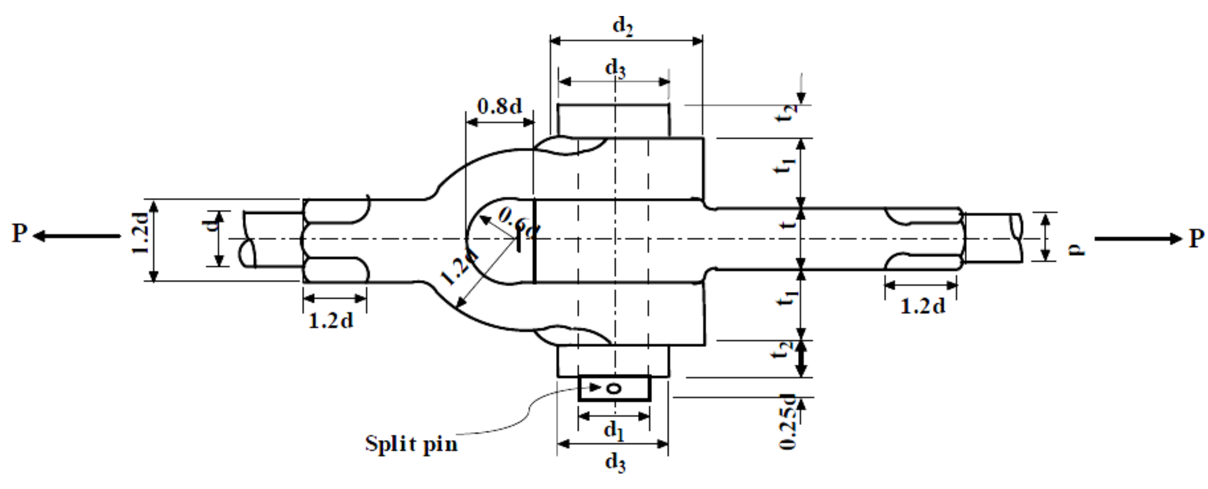


Fig: A typical knuckle joint

These joints are used for different types of connections e.g. tie rods, tension links in bridge structure. In this, one of the rods has an eye at the rod end and the other one is forked with eyes at both the legs. A pin (knuckle pin) is insertedthrough the rod-end eye and fork-end eyes and is secured by a collar and a split pin.

Normally, empirical relations are available to find different dimensions of the joint and they are safe from design point of view. The proportions are given in the figure,

d = diameter of rod

$d_1 = d$ $\quad$ $t = 1.25d$

$d_2 = 2d$ $\quad$ $t_1 = 0.75d$

$d_3 = 1.5.d$ $\quad$ $t_2 = 0.5d$

Mean diameter of the split pin = 0.25 d

However, failures analysis may be carried out for checking. The analyses are shown below assuming the same materials for the rods and pins and the yield stresses in tension, compression and shear are given by ζt, ζc and η.

1. Failure of rod in tension:

$$\frac{\pi}{4} d^2 \sigma_t = P$$

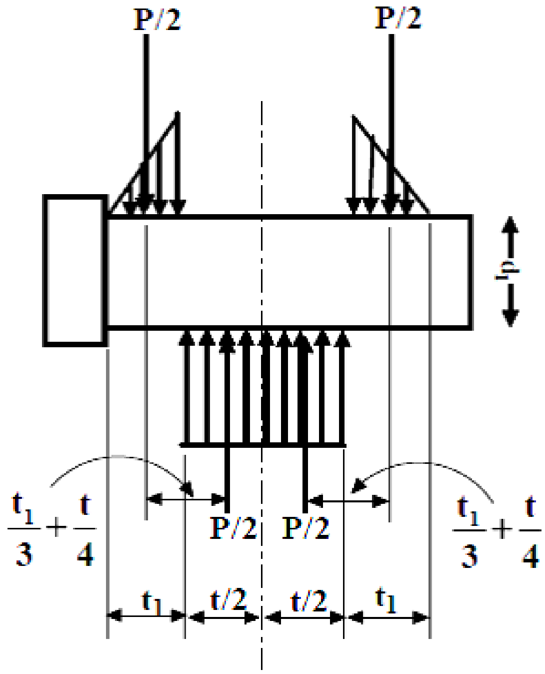
2. Failure of knuckle pin in double shear:

$$2 \frac{\pi}{4} d_1^{\,2} \tau = P$$

3. Failure of knuckle pin in bending (if the pin is loose in the fork) assuming a triangular pressure distribution on the pin, the loading on the pin is shown in figure

Equating the maximum bending stress to tensile or compressive yield stress we have

$$\sigma_t = \frac{16P\left(\frac{t_1}{3} + \frac{t}{4}\right)}{\pi d_1^{\,3}}$$



4. Failure of rod eye in shear:

$$(d_2 - d_1)t\tau = P$$

5. Failure of rod eye in crushing:

$$d_1 t\sigma_c = P$$

6. Failure of rod eye in tension:

$$(d_2 - d_1)t\sigma_t = P$$

7. Failure of forked end in shear:

$$2(d_2 - d_1)t_1\tau = P$$

8. Failure of forked end in tension:

$$2(d_2 - d_1)t_1\sigma_t = P$$

9. Failure of forked end in crushing:

$$2d_1t_1\sigma_c = P$$

The design may be carried out using the empirical proportions and then the analytical relations may be used as checks.

Design a socket and spigot type cotter joint to sustain and axial load of 100 kN. The material selected for the joint has the following design stresses ζT= 100 N/mm2, ζe = 150 N/mm2 and, η = 60 N/mm^2.

Design of Rod $\Rightarrow$ axial Stress $\sigma = \frac{4P}{\pi d^2} \rightarrow 100 = \frac{4 \times 10^5}{\pi d^2} \rightarrow \boxed{d} = 35.68 \simeq 36$m

Design of Spigot and Cotter

E 17-69

Crushing Strength of cotter $F = d_1 t \sigma_c \rightarrow 10^6 = d_1 t(150) \rightarrow \boxed{d_1 t} = 666.67$

E 17.63

Axial Stress across the slot of the rod $\rightarrow \sigma = \frac{4F}{\pi d_1^2 - 4d_1 t} \rightarrow 100 = \frac{4 \times 10^5}{\pi d_1^2 - 4(666.67)}$

$\boxed{d_1} = 46.06$ mm $\simeq \boxed{48 \text{ mm}}$

$48t = 666.67 \rightarrow \boxed{t} = 13.80$ mm $\simeq \boxed{14 \text{ mm}}$

E 17. 65

Shearing strength of cotter $\rightarrow F = 2bt\tau \rightarrow 10^5 = 2b \times 14 \times 60 \rightarrow b = 59.52$

Mean width of cotter by 60 mm

E 17. 66

Shearing Stress due to double shear of the rod end $\rightarrow \tau = \frac{F}{2ad_1} \rightarrow 60 = \frac{10^5}{2a48}$

Rod end distance from slot $\boxed{a} = 17.36 \simeq \boxed{18 \text{ mm}}$

E 17-68 Design of SPIGOT Collar

Bending Stess in the collor $\rightarrow \sigma_r = \frac{4F}{\pi(d_2^2 - d_1^2} \rightarrow 190 = \frac{4 \times 10^5}{\pi(d_2^2 - 48^2)} \rightarrow d_2 = 56.15$ $\boxed{d_2 \simeq 58 \text{ mm}}$

E 17-72

Shear Stress luduced in the coller $\rightarrow \tau = \frac{F}{\pi d_1 e} \rightarrow 60 = \frac{10^5}{\pi \times 48.e} \rightarrow e = 11.05 = 12mm$

E 17-64 **Design of SOCKET** **Thickness of Spigot collor**

Tensile Stress in the socket

$$\sigma = \frac{4F}{\pi(d_3^2 - d_1^2) - 4t(d_3 - d_1)} \rightarrow 100 = \frac{4 \times 10^5}{\pi(d_3^2 - 48^2) - 4 \times 14(d_3 - 48)}$$

E 17-70

Crushing stress incluced us the socket

$$\sigma c = \frac{F}{(d_4 - d_1)t} \rightarrow 150 = \frac{10^4}{(d_4 - 48)14} \rightarrow d_4 = 95.62$$

$$\pi d_3^2 - 7238.23 - 56d_1 + 2688 = 6000$$

$$\boxed{d_4 = 96 \text{ mm}}$$

E 17-67

Shear Stress iuduced in the socket end $\tau = \frac{F}{2c(d_4 - d_1)} \rightarrow 60 = \frac{10^5}{20(96 - 48)}$

Thickness of socket collar C = 17.36 $\simeq$ 18 mm.

E 17-73

Shear Stress in the socket $\tau = \frac{F}{\pi d_1 h} \rightarrow 60 = \frac{10^5}{\pi 48h}$

Thickness of socket at the or d end h = 11.05 $\simeq$ 12 mm.

Design a cotter joint to sustain an axial load of 100 kN. Allowable stress in tension 80 MPa. Allowable stress in compression 120 MPa. Allowable shear stress 60 MPa. Allowable bearing pressure 40 MPa.

1. Design of Rod (E 17.62): Axial stress in the rod}

$$\sigma = \frac{4F}{\pi d^2} \rightarrow 80 = \frac{4 \times 10^5}{\sigma d^2} \rightarrow d = 39.89 \simeq \boxed{d = 40mm}$$

2. Design of spigot and cotter}

i) Crushing strength of cotter (E 17.69)}

$$F = d_1 t \sigma_c \rightarrow 10^5 = d_1 t(120) \rightarrow d_1 t = 833.33$$

ii) Axial stress across the slot of the rod (E 17.63)}

$$\sigma = \frac{4F}{\pi d_1^2 - 4d_1 t} \rightarrow 80 = \frac{4 \times 10^5}{\pi d_1^2 - 4(833.33)} \rightarrow \boxed{d_1 = 51.50mm}$$

using $d_1 t = 833.33$ and d_1 = dia of spigat $= 51.50$, we get

t = thickness of colter $= 16.18mm$

iii) Shearing strength of colter (E 17.65)}

$$F = 2bt\tau \rightarrow 10^5 = 2b(16.18)60 \rightarrow b = 51.50 \simeq \boxed{b = 52mm} (\text{mean}$$

iv) Shearing stress due to double shear at the rod end (E 17.66)

$$\tau = \frac{F}{2ad_1} \rightarrow 60 = \frac{10^5}{2a(51.50)} \rightarrow \boxed{a = 16.18\text{ mm}} \text{(Rod and distance from slot)}$$

3. Design of Spigot collar

i) Bearing stress in the coltar E (17.68)}.

$$\sigma_c = \frac{4F}{\pi(d_2^2 - d_1^2)} \rightarrow 120 = \frac{4 \times 10^5}{\pi\left[d_2^2 - 51.56^2\right]} \rightarrow \boxed{d_2 = 60.94\text{mm}}$$

(Diameter of Spigot collar)

ii) Shear stress induced in the collar (E 17.72)

$$\tau = \frac{F}{\pi d_1 e} \rightarrow 60 = \frac{10^6}{\pi(51.50)e} \rightarrow \boxed{e = 10.30} \text{(Thickness of spigot coller)}$$

4. Design of socket

i) Tensile stress in the socket (E 17.64)}

$$\sigma = \frac{4F}{\pi(d_3^2 - d_1^2) - 4t(d_3 - d_1)} \rightarrow 100 = \frac{4 \times 10^5}{\pi(d_3^2 - 51.50^2) - 4(16.18)(d_3 - 51.50)}$$

we ger quadratic equ $d_3^2 - 20.60, d_3 - 2864.54 = 0;$ $\boxed{d_3 = 65\text{mm}}$ outside dia of socket

ii) Crushing stress induced in the socket (E 17.70)}

$$\sigma_c = \frac{F}{(d_4 - d_1)t} \rightarrow 120 = \frac{10^5}{(d_4 - 51.50)16.18} \rightarrow \boxed{d_4 = 103.25\text{mm}} \text{ Dia of socket collar}$$

iii) Shearing stress induced in the socket end (E 17. 67)}

$$\tau = \frac{F}{2c(d_4 - d_1)} \rightarrow \boxed{C = 16.10} \text{ thickness of socket collar}$$

$$(\tau = 60; F = 10^5; d_4 = 103.25; d_1 = 51.52)$$

iv) Shear stress in the socket (E 17.73)

$$\tau = \frac{F}{\pi d_1 h} \rightarrow 60 = \frac{10^5}{\pi(51.50)h} \rightarrow \boxed{h = 10.30\text{mm}} \text{Thickness of socket at the rod end}$$

5. Bending stress induced in the cotter (E 17.74)}

$$\sigma_{b(max)} = F\left[\frac{d_1 + 2d_4}{4tb^2}\right] = 10^5\left[\frac{51.50 + 2(103.25)}{4 \times 16.18 \times 52^2}\right] = 147.43\text{MPa}$$

(To reduce σ_b value we can increase t or b)

Design a Knuckle joint to transmit 150 kN. The design stresses may be taken as 75 N/mm^2 in tension, 60 N/mm^2 in shear and 150 N/mm^2 in compression.

i) Design of rod & rod end

* Tensile stress in the rod

$$\sigma_t = \frac{4F}{\pi d^2}$$

$$75 = \frac{4 \times 150 \times 10^3}{\pi d^2}$$

$\therefore$ Diameter of rod d = 50.46mm ~ 52mm

* Diameter of rod end d1=1.2d=1.2×52=62.4mm

ii) Design of knuckle pin & collar

*Design of knuckle pin d2 = d = 52 mm

*Dia of pin head = Dia of collar

d3=1.5d = 1.5×52 = 78 mm

*Thickness of pin head = Thickness of collar

h=0.5d = 0.5×52=26 mm

*As the knuckle pin is subjected to double shear shear strength of knuckle pin

$$F = 2 \times \frac{\pi}{4} di^2 \times \tau$$

$$150 \times 10^3 = 2 \times \frac{\pi}{4} \times 52^2 \times \tau$$

$\therefore$ Shear stress induced τ =35.315 N/mm² < 60 N/mm² in the knuckle joint

$\therefore$ The design of knuckle pin is safe

iii) Design of eye

*Thickness of eye b = 1.25d = 1.25×52=65mm

*Diameter of eye d4 = 2d = 2×52 = 104 mm

*Shear strength of eye F = τb(d4-d2)

$\therefore$ Shear stress induced in the eye

$$\tau = \frac{F}{b(d_4 - d_2)} = \frac{150 \times 10^3}{65(104-52)} = 44.4 N/mm^2 < 60 N/mm^2$$

*Tearing strength of eye F= σ_t b(d4-d2)

$\therefore$ Tensile stress induced in the eye

$$\sigma_t = \frac{F}{b(d_4 - d_2)} = \frac{150\times10^3}{65(104-52)} = 44.4N/mm^2 < 75N/mm^2$$

*Crushing strength of eye F = d2b σ_c

∴ Compressive stress induced in the eye

$$\sigma_c = \frac{F}{d_2 b} = \frac{150\times10^3}{52\times65} = 44.4N/mm^2 < 130N/mm^2$$

The design of eye is safe.

iv) Design of fork

*Thickness of fork a=0.75 d = 0.75 ×52 = 39 mm

*Shear strength of fork F = 2aτ(d4-d2)

∴ shear stress induced in the fork

$$\tau = \frac{F}{2a(d_4 - d_2)} = \frac{150\times10^3}{2\times39\times(104-52)} = 36.98N/mm^2 < 60N/mm^2$$

*Tearing strength of fork F = 2a (d4-d2) σt

∴ Tensile stress induced in the fork

$$\sigma_t = \frac{F}{2a(d_4 - d_2)} = \frac{150\times10^3}{2\times39\times(104-52)} = 36.98N/mm^2 < 75N/mm^2$$

*Crushing strength of fork F= 2ad2σc

Compressive stress induced in the fork

$$\sigma_t = \frac{F}{2ad_2} = \frac{150\times10^3}{2\times39\times52} = 36.98N/mm^2 < 130N/mm^2$$

∴ The design of fork is safe.

KEYS AND COUPLINGS

INTRODUCTION

A key is a piece of steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Objectives

After studying this unit, you should be able to

- Identify keys and their application,
- Calculate forces on keys, and
- Design keys.

TYPES OF KEYS

The following types of keys are important from the subject point of view :

(a) Shunk keys,

(b) Saddle keys,

(c) Tangent keys,

(d) Round keys, and

(e) Splines.

We shall now discuss the above types of keys, in detail, in the following sections.

Sunk Keys

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley or gear. The sunk keys are of the following types :

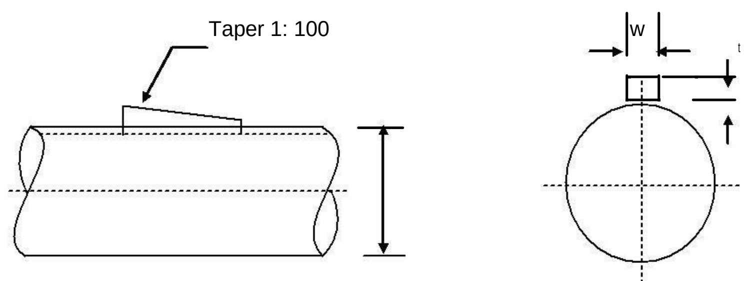
Rectangular Sunk Key

A rectangular sunk key is shown in Figure 6.1. The usual proportions of this key are :

Width of key, $w = \frac{d}{4}$; and thickness of key, $t = \frac{2w}{3} = \frac{d}{6}$

where d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only.



Square Sunk Key

The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e.

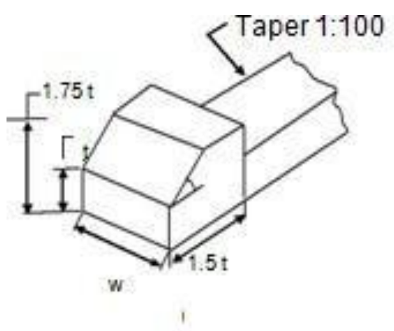
$$w = t = \frac{d}{4}$$

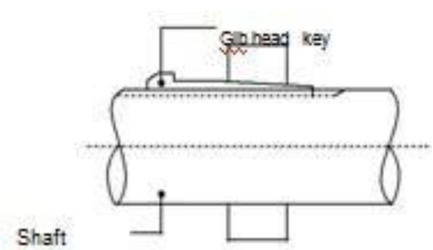
Parallel Sunk Key

The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating part is required to slide along the shaft.

Gib-head Key

It is a rectangular sunk key with a head at one end known as **gib head**. It is usually provided to facilitate the removal of key. A gib head key is shown in Figure and its use in shown in Figure.





The usual proportions of the gib head key are :

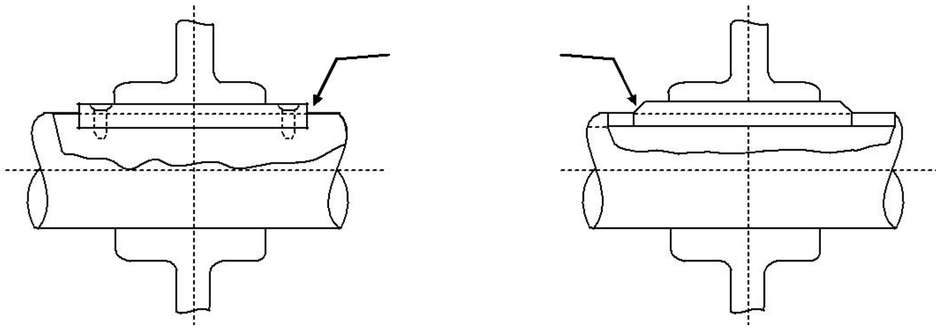
Width, $w = \frac{d}{4}$;

and thickness at large end, $t = \frac{2w}{3} = \frac{d}{6}$.

Feather Key

A key attached to one member of a pair and which permits relative axial movement of the other is known as **feather key**. It is a special key of parallel type which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.

The feather key may be screwed to the shaft as shown in Figure or it may have double gib heads as shown in Figure. The various proportions of a feather key are same as those of rectangular sunk key and gib head key.



Feather Key

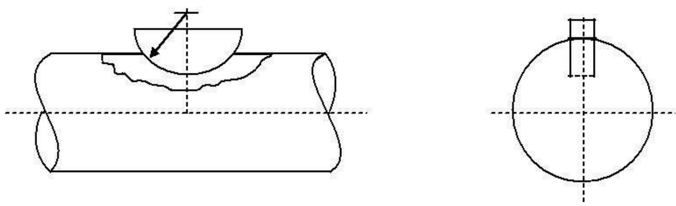
The following Table shows the proportions of standard parallel, tapered and gib head keys, according to IS : 2292 and 2293-1974 (Reaffirmed 1992).

Proportions of Standard Parallel, Tapered and Gib Head Key

Shaft Diameter (mm) upto and Including	Key Cross-section		Shaft Diameter (mm) upto and Including	Key Cross-section	
	Width (mm)	Thickness (mm)		Width (mm)	Thickness (mm)
6	2	2	85	25	14
8	3	3	95	28	16
10	4	4	110	32	18
12	5	5	130	36	20
17	6	6	150	40	22
22	8	7	170	45	25
30	10	8	200	50	28
38	12	8	230	56	32
44	14	9	260	63	32
50	16	10	290	70	36
58	18	11	330	80	40
65	20	12	380	90	45
75	22	14	440	100	50

Woodruff Key

The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Figure. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.



Woodruff Key

The main advantages of a woodruff key are as follows :

(c) It accommodates itself to any taper in the hub or boss of the mating piece.

(d) It is useful on tapering shaft ends. Its extra depth in the shaft prevents any tendency to turn over in its keyway.

The disadvantages are :

(a) The depth of the keyway weakens the shaft.

(b) It can not be used as a feather.

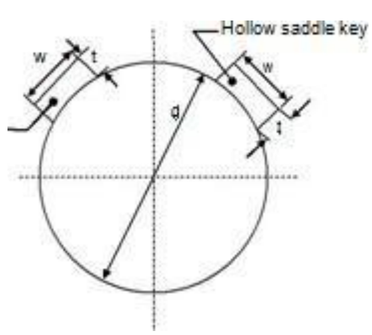
Saddle Keys

The saddle keys are of the following two types :

Flat saddle key, and

Hollow saddle key.

A **flat saddle key** is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in Figure. It is likely to slip round the shaft under load. Therefore, it is used for comparatively light

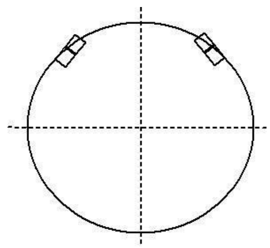


Saddle Key

A **hollow saddle key** is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore, these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams, etc.

Tangent Keys

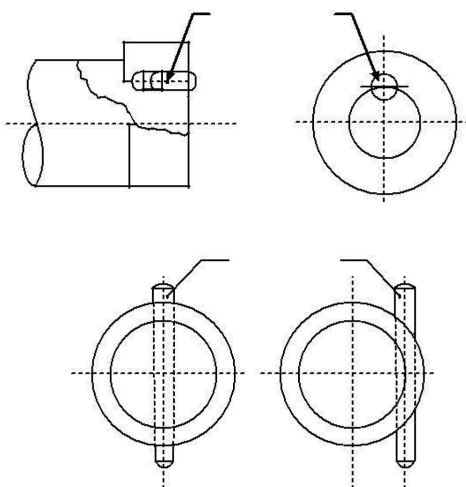
The tangent keys are fitted in pair at right angles as shown in Figure. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.



Tangent Keys

Round Keys

The round keys, as shown in Figure, are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage of manufacturing as their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.



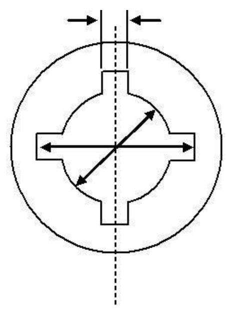
Round Keys

Sometimes the tapered pin, as shown in Figure, is held in place by the friction between the pin and the reamed tapered holes

Splines

Sometimes, keys are made integral with the shaft which fit in the keyways broached in the hub. Such shafts are known as **splined shafts** as shown in Figure. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.

The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive.



FORCE ACTING ON A SUNK KEY

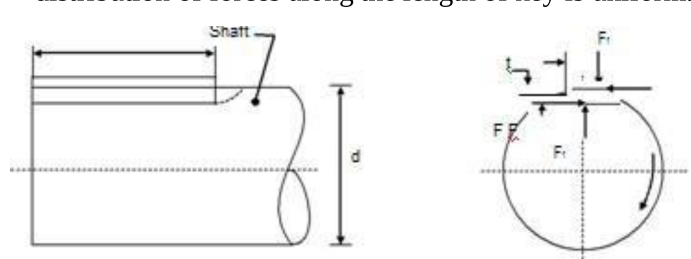
When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

(a) Forces (F1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

(b) Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub.

The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Figure. In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.



STRENGTH OF A SUNK KEY

A key connecting the shaft and hub is shown in Figure.

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key,

t = Thickness of key, and

τ and σ_c = Shear and crushing stresses for the material of key

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

$$F = \text{Area resisting shearing} \times \text{Shearing stress} = l \times w \times \tau$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2}$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

∴ Torque transmitted by the shaft,

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

Problems:

If a shaft and key are made of same material, determine the length of the key required in terms of shaft diameter, taking key width and key thickness. Assume keyway factor as 0.75.

Given data : $b = \frac{d}{4}$; $h = \frac{3d}{16}$; Torque transmitted by the shaft

$$M_t = \frac{\pi d^3 n \tau_s}{16} = \frac{0.75 \pi d^3 \tau s}{16}$$

By considering shear resis tan ce of the key ; $M_t = \frac{bld}{2}\tau k$

$$\text{i.e., } M_t = \frac{d^2L\tau_k}{8}$$

By equation both

$$\frac{0.75\pi d^3\tau_s}{16} = \frac{d^2L\tau_k}{8}$$

$\therefore$ Length of the key $L = 1.178d \rightarrow (1) \because (\tau_k = \tau_s)$

Considering the oweshing resis tan ce of the key

$$M_t = \frac{hld\sigma_c}{4}$$

$$\text{i.e., } M_t = \frac{3d}{16} \times \frac{Ld}{4} \times 2\tau_k \left(\because \sigma_c = 2\tau_k\right)$$

Equation both, we have $\frac{0.75\pi d^3\tau_s}{16} = \frac{3d}{16} \times \frac{Ld}{4} \times 2\tau_K$

$$\therefore L = 1.5708d \rightarrow (2)$$

Taking the greater value among both, values

$$L = 1.5708d$$

Design a rigid flange coupling to transmit 18kW at 1440 rpm. The allowable shear stress in the cast iron flange is 4 MPa. The shaft and keys are made of AISI 1040 annealed steel with ultimate strength and yield stress valued as 518.8 MPa and 353.4 MPa, respectively. Use ASME code to design the shaft and the key.

Given N = 18kW, n = 1440rpm, σ_{all} =4MPa, σ_{wit} = 518.8 MPa - shaft σ_{wit} =353.4 MPa far key:

1. Tarque Transmitter :

$$M_t = 9550 \times \frac{N}{h} \times 1000 = 9550 \times \frac{18}{1440} \times 1000 = 119375\,N-mm$$

2. Shaft dia:

$$M_t = \frac{\pi}{16} d^3 n \tau_s$$

$$119375 = \frac{\pi}{16} d^3 \times 0.75 \times 129.7$$

$$d = 19mm$$

adopt dimension of 22mm or 20mm

Let us select 22mm

4. Bolt circle diameter

$D_1 = 2d + 50 = 2\times 22 + 50 = 94mm$

5. Design of Hub:

i) Hub dis $D_2 = 1.5d + 25 = (1.5\times22) + 25 = 58mm$

ii) Length of hub L = 1.25d + 18.75 = 46.25mm

6. Design of Flanga :

(i) Outer dia of flange D = 2.5d + 75 = 130mm

(ii) Thickness of flange t = 0.5d = 11mm

(iii) Check far the flange

$$M_t = t(\pi D_2)\tau_F\left(\frac{D_2}{2}\right)$$

$$19375 = 11\times(\pi\times50)\tau_F\left(\frac{50}{2}\right)$$

$$\tau_{F\,induced} = 2.76\,N/mm^2$$

Induced stress is lesser than allowable stress of 4 N/mm² hence safe

7. Design of bolts

(i) No of bolts $i = 0.02d + 3 = 0.02\times 22 + 3 = 3.44$

$$i = 4$$

(ii) Dia of bolt $M_t = i\left(\frac{\pi d_1^{\,2}}{4}\right)\times\tau_b\left(\frac{D_1}{2}\right)$

$$d_1 = 5.5mm$$

Hence adopt $M_{7\times1}$ with $A_c = 28.9mm^2$

(iii) Gushing strength of bolt

$$M_t = id_2 t(\sigma_c)_b \frac{D_1}{2}$$

$$(\sigma_c)_b = \frac{119375 \times 2}{4 \times 5.5 \times 11 \times 94} = 10.5 \text{ N/mm}^2$$

$$(\sigma_c)_{all} = 152 \text{ N/mm}^2$$

$(\sigma_c)_{induce} < \sigma_c$ allowable hence design is safe.

8. Design of key:

Adopt 6 × 6 taper key with length = 50mm

Design a cast iron flanged couplings for a steel shaft transmitting 100 kW at 250 rpm. Take the allowable shear stress for the shaft as 40 N/mm^2. The angle of twist is not to exceed 1° in a length of 20 diameters. Allowable shear stress for the bolts is 13 N/mm^2. The allowable shear stress in the flange is 14 N/mm^2. For the key shear stress is 40 N/mm^2 and compressive stress is 80 N/mm^2.

$$M_t = \frac{9550N}{n} = \frac{9550 \times 100}{250} = 3820Nm = 38,20,000 \text{ N mm-} \quad \text{E -19-3}$$

Considering Shear,

$$M_t = \frac{\pi}{16} d^3 \tau_s \Rightarrow 3820000 = \frac{\pi}{2} d^3 (40) \rightarrow d = 78.64mm \quad \text{E 19.2}$$

Considering rigidity,

$$M_t = \frac{\theta d^4 G}{L584} \Rightarrow 3820000 = \left(\frac{1}{25d}\right) \frac{d^4 (80000)}{584} d = 88.67 \simeq 90mm$$

[assume G = 80 GPa]

Bolt circle diameter; $D_1 = 2d + 50 = 2(90) + 50 = 230$ mm — E 19.12 b

Hub Diameter ; D_2 = 1-5d + 25 = 1.5 (90) + 25 = 160 mm — E 19.13 b

Length of Hub ; L = 1.25d + 18.75= 1.25 (90) + 18.75 = 131. 25 mm — E 19.14 d

Outer Dia of flange ; D = 2.5d + 75 = 2.5 (90) + 75 = 300 mm — E 19.14 b

Thickness of flange ; t = 0.5d = 0.5 (90) = 45 mm — E 19-6

Check for flange ; $M_t = t \pi D_2 \tau_f \left(\frac{D_2}{2}\right)$ or

$$\tau_{f(induced)} = \frac{2M_t}{t\pi D_2^2} = \frac{2 \times 3820000}{45 \times \pi \times 160^2} = 2.11 Mpa < 14 MPa \therefore\ safe$$

No of Both; i = 0.02d + 3 = 0.02 (90) + 3 = 3 $\simeq$ 4 numbers E 19. 1 b

diameter of bolt (d1) ; $M_t = \frac{\pi (d_1)^2}{4} \tau_b \frac{D_1}{2} \Rightarrow 38,20,000 = \frac{4 \times \pi d_1^2 \times 13 \times}{4} \frac{230}{2}$

d_1=28.52 $\simeq$ 29 mm

Checking for Dia of bolts ; Mt= $i d_1 t (\sigma_a)_{bolt} \frac{D_1}{2}$ E 19.5

$$(\sigma_{crushing})_{bolt} = \frac{2M_t}{i d_1 t D_1} = \frac{2 \times 3820000}{4 \times 29 \times 45 \times 230} = 6.36 \text{ MPa (safer)}$$

Design of Key : assume SUNK - key

Select taper 1 in 100

refer T 17.4 upto 95 mm width, b = 28 mm ; h= 18 mm

T 17-5 L = 140 ; L>hublength.

Check for key b = $\frac{2M_t}{\tau_{d_2} Ld}$ $E19.50$

Shear Induced $\tau_{d_2} = \frac{2M_t}{bLd} = \frac{2 \times 3820000}{28 \times 140 \times 90} = 21.66 MPa$

allowable crushing stress = 2 τ_{d2}=2 x shear stress in key = 2 x 40 = 80 MPa

Thickness of key h = $\frac{4M_t}{\sigma_b^1 Ld} \Rightarrow \sigma_b^1 = \frac{hld}{4m_t} = \frac{18 \times 140 \times 90}{4 \times 3820000} = 0.015$ E 19.51

normally σ_h^1=2 τ_{d_2} = 80 MPa.

Design a flanged coupling to connect the shafts of motor and pump transmitting 15 kW power at 600 rpm. Select C40 steel for shaft and C35 steel for bolts, with factor of safety = 2.
Use allowable shear stress for Cast-Iron flanges =15 N/mm^2 ζ =162 N/mm^2; and ζ = 81 N/mm^2 for bolts (J = 152 N/mm^2 and η = 76 N/mm^2.

C40 – shafts(MDDH T1.5) $\sigma_{tensile} = 568.8 - 666.8\text{MPa}; \sigma_y = 328.6\text{MPa}$

(assume for keys also C40) $\tau_y = 0.5\,\sigma_y = \dfrac{328.6}{2} = 164.3\text{MPa}$

$$\tau_{d2} = \tau_{ed} = \frac{\tau_m}{fos} = \frac{164.3}{2} = 82.15\text{MPa}$$

Crushing stress in key $\sigma_b^1 = 2\tau_{d2} = 2(82.15) = 164.3\text{MPa}$

C35 – bolt(T 1.5 MDDH) $\sigma_y = 304\text{MPa}$

$$\tau_y = \frac{304}{2} = 152\text{N/mm}^2$$

$$\tau_b = \frac{152}{FOS} = \frac{152}{2} = 76\,\text{N/mm}^2$$

Torque to be transmitted; $M_t = \dfrac{9550 \times 1000 \times N}{n} = \dfrac{9550 \times 1000 \times 15}{600} = 238,750\ \text{Nmm}$

E19.2 Diameter of shaft; use relations $M_t = \dfrac{\pi}{16} d^3 \times \eta\tau_s$
(assume 100% η)

$$238,750 = \frac{\pi}{16} \times d^3 \times \frac{100}{100} \times 28.15 \Rightarrow d = 24.52\text{mm}$$

E19.12b Boltcircle diameter $D_1 = 2d + 50 = 2(30 + 50) = 110\text{mm}$

E19.13b Design of Hub dia $D_2 = 1.5d + 25 = 1.5(30) + 25 = 70\text{mm}$

E19.14d length of hub $L = 1.25d + 18.75 = 1.25(30) + 18.75 = 56.25\text{mm}$

E19.13b Design og flange octes dia $D = 2.5d + 75 = 2.5(30) + 75 = 150\text{mm}$

Flange thickness $t = 0.5d = 0.5(30) = 15\text{mm}$

Check τ_{ind} in *flange*; E19.6 $M_t = t(\pi D_2)\tau_f\left(\dfrac{D_2}{2}\right) \Rightarrow 238.750 = 15(\pi 70)\tau_{fi}\left(\dfrac{70}{2}\right)$

τ_{ind} in *flange*; $= 2.07\text{MPa}$, ... So, O.K

E19.1b No of bolts $i = 0.0200d + 3 = 0.02(30) + 3 = 3.6 \simeq 4$

E19.4 Dia of bolts, using $M_t = i\dfrac{\pi d_1^2}{4}\tau_b\dfrac{D_1}{2} \Rightarrow 238,750 = 4\dfrac{\pi d_1^2}{4}(76)\dfrac{110}{2} \Rightarrow d_1 = 4.26\text{mm}$

For $d_1 = 4.26$ from T 18.7, select major Dia = 5mm, Pitch = 0.5 i.e., M5 × 0.5, min minor Dia 4.386565

E19.5 Check for bolt; $M_t = id_1t(\sigma_c)_b\dfrac{D_1}{2} \Rightarrow (\sigma_c)_{b\,induced} = \dfrac{2Mt}{id_1tD_1} = \dfrac{2(238,750)}{4(4.386565)15 \times 110}$

$$= 16.50\text{MPa}$$

$$(\sigma_c)_{b\,all} = 2\tau_b = 2(76) = 152\text{MPa} \quad \therefore \text{safe}$$

T17.4 Select sunk key with taper 1:100 for d = 30mm, width (b) = 8, h(height) = 7mm

Preferred length Use T 17 – 5(for b = 8; h = 7 and L = 56.25) = 63mm

E19.50 Check for key; $b = \frac{2Mt}{\tau_{d2} l_d} \rightarrow 8 = \frac{2 \times 238,750}{\tau d_2 (63)30} \rightarrow \tau_{d2} = 31.60\text{MPa} < 82.5\text{MPa}, \therefore \text{safer}$

E19.51 $h = \frac{4M_t}{\sigma_b l_d} \rightarrow 7 = \frac{4 \times 238,750}{\sigma_b^I (65)30} \rightarrow \sigma_b^I = 69.69\text{MPa} < 164.3\text{MPa} \therefore \text{safer}$

A splined connection in an automobile transmission consists of 10 splines cut in a 58 mm diameter shaft. The height of each spline is 5.5 mm and the keyways in the hub are 45 mm long. Determine the power that may be transmitted at 2500 rev/min if the allowable normal pressure on the splines is limited to 4.8 MPa.

i=10 D=58mm h=5.5mm l=45mm

n=2500rpm p=4.8MPa = 4.8N/mm^2

Torque transmitted by the spline shaft

$$M_t = \frac{1}{2} p.h.l.i(D - h)$$

$$= \frac{1}{2} \times 4.8 \times 5.5 \times 45 \times 10(58 - 5.5)$$

$$M_t = 324843.75\,\text{N} - \text{mm}$$

Also

$$M_t = 9550 \times 1000 \times \frac{N}{n}$$

$$\text{i.e.,}\ 324843.75 = 9550 \times 1000 \times \frac{N}{250}$$

$$\therefore N = 85.04\,\text{KW}$$

UNIT 7

RIVETED AND WELDED JOINTS

Instructional Objectives

- *Basic types of riveted joints.*
- *Different important design parameters of a riveted joint.*
- *Uses of riveted joints.*
- *Basic failure mechanisms of riveted joints.*
- *Concepts of design of a riveted joint.*
- *Different types of welded joints.*
- *Factors that affect strength of a welded joint.*
- *Possible failure mechanisms in welded joints.*
- *How to design various kinds of welding joints.*

Introduction

In engineering practice it is often required that two sheets or plates are joined together and carry the load in such ways that the joint is loaded. Many times such joints are required to be leak proof so that gas contained inside is not allowed to escape. A riveted joint is easily conceived between two plates overlapping at edges, making holes through thickness of both, passing the stem of rivet through holes and creating the head at the end of the stem on the other side. A number of rivets may pass through the row of holes, which are uniformly distributed along the edges of the plate. With such a joint having been created between two plates, they cannot be pulled apart. If force at each of the free edges is applied for pulling the plate apart the tensile stress in the plate along the row of rivet hole and shearing stress in rivets will create resisting force. Such joints have been used in structures, boilers and ships.

The development of welding technology in 1940s has considerably reduced the riveted joint applications. Welding is the method of locally melting the metals (sheets or plates–overlapping/butting) with intensive heating along with a filler metal or without it and allowing cooling them to form a coherent mass, thus creating a joint. Such joints can be created to make structures, boilers, pressure vessels, etc. and are more conveniently made in steel. The progress has been made in welding several types of steels but large structure size may impede the use of automatic techniques and heat treatment which becomes necessary in some cases. Welded ships were made in large size and large number during Second World War and failures of many of them spurted research efforts to make welding a better technology.

Head Forming

You know that the riveted joint is created by passing the stem of a rivet through holes in two plates as is shown in Figure 3.1(a). The creation of head by process of upsetting is shown in Figure 3.1(b). The upsetting of the cylindrical portion of the rivet can be done cold or hot. When diameter of rivet is 12 mm or less, cold upsetting can be done. For larger diameters the rivet is first heated to light red and inserted. The head forming immediately follows. The rivet completely fills the hole in hot process. Yet it must be understood that due to subsequent cooling the length reduces and diameter decreases. The reduction of length pulls the heads of rivet against plates and makes the joint slightly stronger. The reduction of diameter creates clearance between the inside of the hole and the rivet. Such decrease in length and diameter does not occur in cold worked rivet.

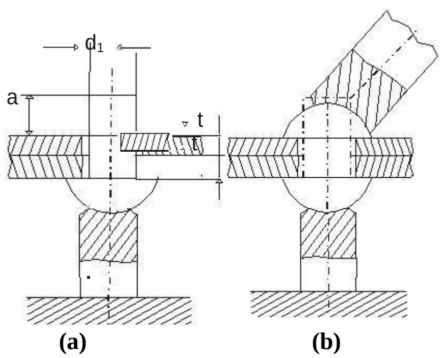
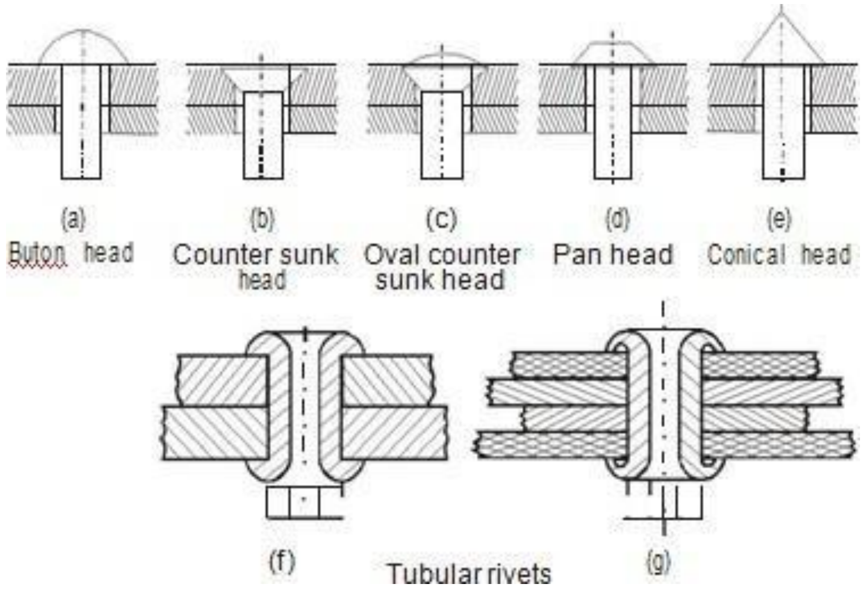


Figure 3.1 : Typical Head Forming of Rivet

Types of Rivets

For steel plates the rivets are normally made in low carbon steel. However, the rivets in copper add to resistance against corrosion and aluminum rivets can be used to reduce the overall weight of the structure. The low carbon steel is standardized in composition particularly for boiler applications.

Rivets with counter sunk head as in Figure 3.2(b) and oval counter sunk rivets shown in Figure 3.2(c) are not as strong as button head rivets. They are used only when protruding rivet heads are objectionable. Pan heads and conical heads, Figures 3.2(d) and (e) are less frequently used and are difficult to form. Tubular rivets, Figures 3.2(f) and (g) are special deviation from solid rivet shank. These rivets are used in aircrafts.



Types of Riveted Joints

The classification of riveted joints is based on following:

a) According to purpose,

b) According to position of plates connected, and

c) According to arrangement of rivets. According to purpose the riveted joints are classified as :

Strong Joints

In these joints strength is the only criterion. Joints in engineering structure such as beams, trusses and machine frames are strong joints.

Tight Joints

These joints provide strength as well as are leak proof against low pressures. Joints in reservoirs, containers and tanks fall under this group.

Strong Tight Joints

These are joints applied in boilers and pressure vessels and ensure both strength and leak proofness.

This classification has no sound basis and is arbitrary. However, it helps understand the basis of design and manufacturing. The hot working of rivets is one-way of making intimate contact between plates in the areas of joint. Further, the holes are drilled and reamed to required tolerances and burrs removed for good contact before rivets are placed in the holes. The edge of the plate is upset by means of a hammer and a caulking tool so that edge is strongly pressed against the plate surface to help leak proofing (Figure 3.3).

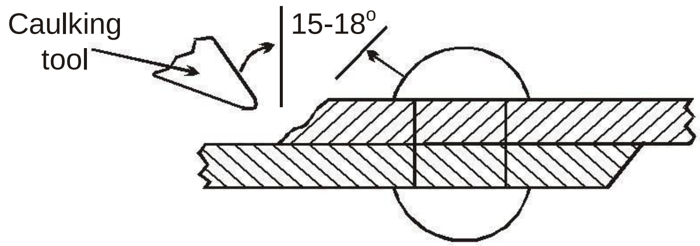
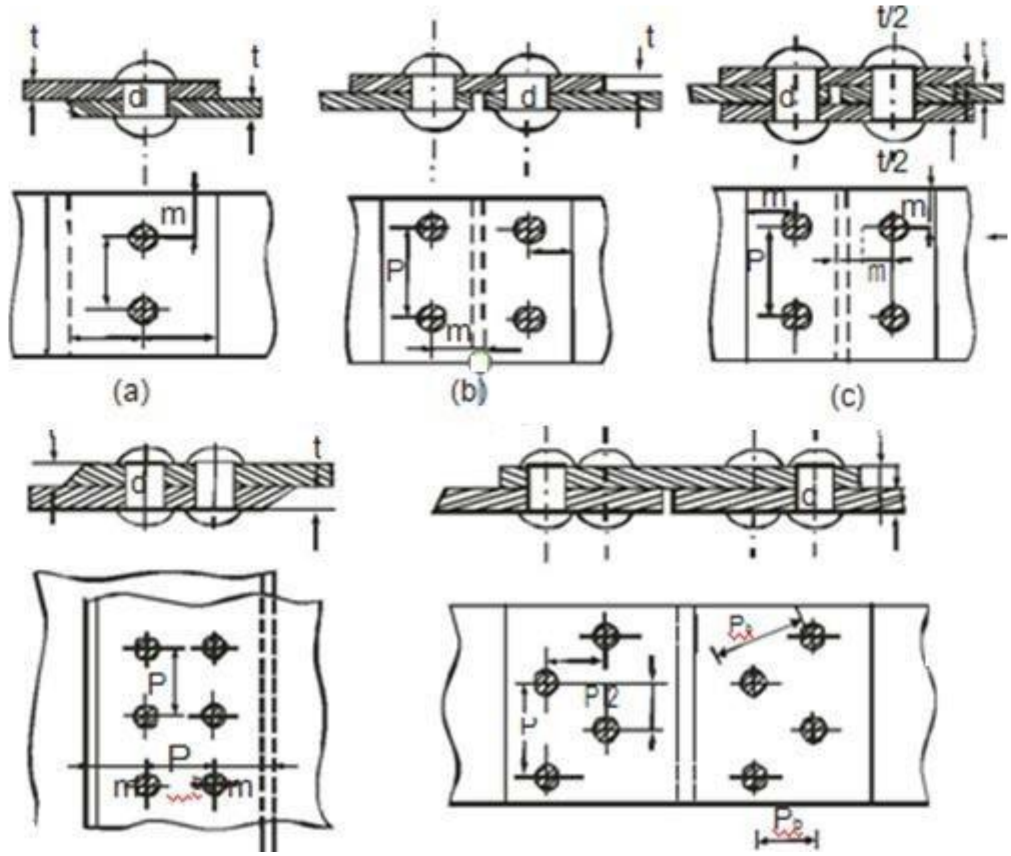


Figure 3.3 : Caulking of Riveted Joint

The riveted joints are classified as (i) **lap joint** and (ii) **butt joint** according to position of plates. In a lap joint the edges of plates are simply laid over each other and riveted. Figures 3.4(a) and (d) show lap joints. If we pull the plates by application of tensile forces, they do not fall in the same line and hence cause the rivets and plates to bend. Plates placed end-to-end and jointed through cover plates form **single cover butt joint**. Such joints are shown in Figures 3.4(b) and (e). You can see that pulling plates apart by collinear tensile forces may still cause bending of rivets. Figures 3.4(c) and (f) show the butting plates covered by two straps and then riveted. Such joints are called **double cover butt joint**. Plate bending and rivet bending are eliminated.

According to arrangement of rivets, the joints are called **single riveted**, (Figures 3.4(a), (b) and (c)) It may be noted that in a single riveted lap joint there is only one row of rivets passing through both plates while in a single riveted butt joint either of single cover or double cover type one row of rivets will pass through each of the plates.

Similarly as shown in Figures 3.4(d) and (e) when two rows of rivets pass through both plates of lap joint it is called **double riveted** lap joint and two rows of rivets pass through each of butting plates the joint is a double riveted single cover butt joint. A double riveted double cover butt joint is shown in Figure 3.4(f).



The arrangement of rivets in Figure 3.4(d) can be described that in both the rows the rivets are opposite to each other while in Figure 3.4(e) the rivets in the adjacent rows are staggered. The joint in Figure 3.4(d) is said to be **chain riveted** while that in Figure 3.4(e) is **zig-zag riveted** joint. In zig-zag riveting the rivet in one row is placed at the middle level of the two rivets in the adjacent row.

Nomenclature

Several dimensions become obviously important in a riveted joint and a design will consist in calculating many of them. These dimensions and their notations as to be used in this text are described below.

Pitch

As seen from Figures 3.4(a), (b) and (c) pitch, denoted by p, is the center distance between two adjacent rivet holes in a row.

Back Pitch

The center distance between two adjacent rows of rivets is defined as back pitch. It is denoted by p_b and is shown in Figures 3.4(d) and (e).

Diagonal Pitch

The smallest distance between centres of two rivet holes in adjacent rows of a zig-zag riveted joint is called diagonal pitch. Denoted by p_d, the diagonal pitch is shown in Figure 3.4(e).

Margin

It is the distance between centre of a rivet hole and nearest edge of the plate. It is denoted by m as shown in Figures 3.4(b), (c) and (d).

The plates to be jointed are often of the same thickness and their thickness is denoted by t. However, if the thicknesses are different, the lower one will be denoted by t_1. The thickness of the cover plate (also known as strap) in a butt joint will be denoted as t_c.

The rivet hole diameter is denoted by d. This diameter is normally large than the diameter of the rivet shank which is denoted by d_1.

A problem of designing of a riveted joint involves determinations of p, p_b, p_d, m, t, t_c and d, depending upon type of the joint.

Modes of Failure of a Riveted Joint

A riveted joint may fail in several ways but the failure occurs as soon as failure takes place in any one mode. Following is the description of modes of failures of a riveted joint. These modes are described with the help of a single riveted lap joint, which is subjected to tensile load P. In general the description will be applicable to any other type of joint. Reference is made in Figure 3.5 in which a single riveted lap joint is shown loaded.

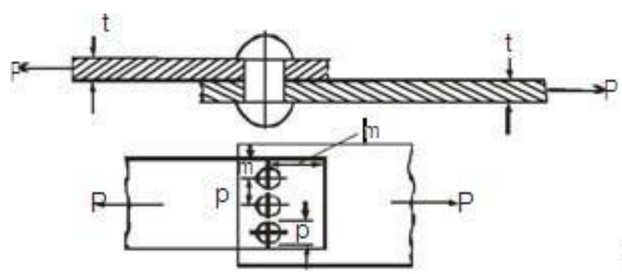


Figure 3.5 : Single Riveted Lap Joint

(a) Tearing of Plate at the Section Weakened by Holes

Figure 3.6 shows this mode of failure. The plate at any other section is obviously stronger, and hence does not fail. If tensile force P is to cause tearing, it will occur along weakest section, which carries the row of rivets. If only one pitch length p is considered; it is weakened by one hole diameter d. The area that resists the tensile force is

$$A_t = (p - d)\ t$$

If the permissible stress for plate in tension is σ_t, then tensile strength of the joint or tensile load carrying capacity of the joint

$$P_t = \sigma_t\ (\ p - d\ )\ t \qquad \text{. . . (3.1)}$$

If P is the applied tensile force per pitch length then the joint will not fail if

$$P \geq P \qquad \text{. . . (3.2)}$$

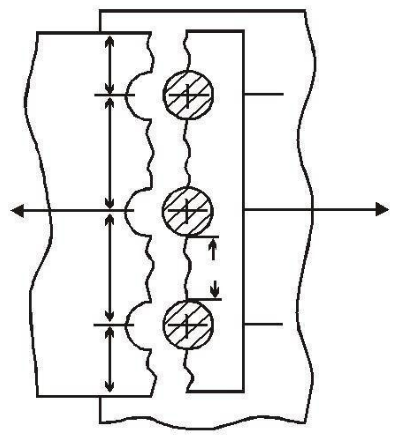


Figure 3.6 : Tearing of Plate at the Section Weakened by Holes

(b) Shearing of Rivet

Figure 3.7 shows how a rivet can shear. The failure will occur when all the rivets in a row shear off simultaneously. Considers the strength provided by the rivet against this mode of failure, one consider number of rivets in a pitch length which is obviously one. Further, in a lap joint failure due to shear may occur only along one section of rivet as shown in Figure 3.7(a). However, in case of double cover butt joint failure may take place along two sections in the manner shown in Figure 3.7(b). So in case of single shear th0e area resisting shearing of a rivet, $A_s = \pi\ d^2/4$

(Since the difference between diameter of hole and diameter of rivet is very small, diameter of hole is used for diameter of the rivet).

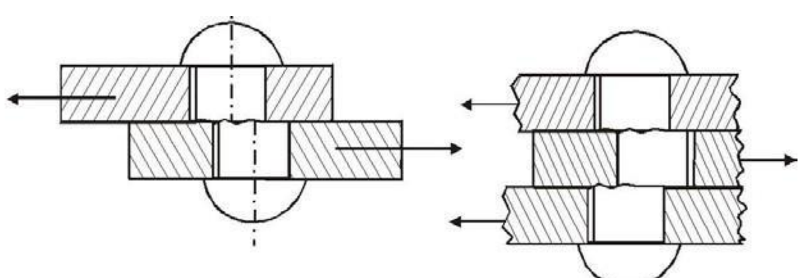


Figure 3.7 : Shearing of Rivet : (a) Single Shear; and (b) Double Shear

If permissible shearing stress in single shear of rivet is τ_s, then the shearing strength or shearing load carrying capacity of the joint.

$$P_s = \tau_s \frac{\pi}{4} d^2 \qquad \ldots (3.3)$$

The failure will not occur if

$$P_S \geq P \qquad \ldots (3.4)$$

We may also write if *n* is the number of rivets per pitch length,

$$P_s = n\, \tau_s \frac{\pi}{4} d^2 \qquad \ldots (3.5)$$

If the rivet is in double shear as in Figure 3.7(b) the effective area over which failure occurs in 2 A_s. The permissible stress in double shear is 1.75 times that in single shear. Hence in double shear

$$P_s = n \times 1.75\, \tau_s \frac{\pi}{4} d^2 \qquad \ldots (3.6)$$

(e) Crushing of Plate and Rivet

Due to rivet being compressed against the inner surface of the hole, there is a possibility that either the rivet or the hole surface may be crushed. The area, which resists this action, is the projected area of hole or rivet on diametral plane. The area per rivet is (see Figure 3.8).

$A_C = dt$. If permissible crushing or bearing stress of rivet or plate is σ_c the crushing strength of the joint or load carrying capacity of the joint against crushing is,

$$P_c = dt\, \sigma_c \qquad \ldots (3.7)$$

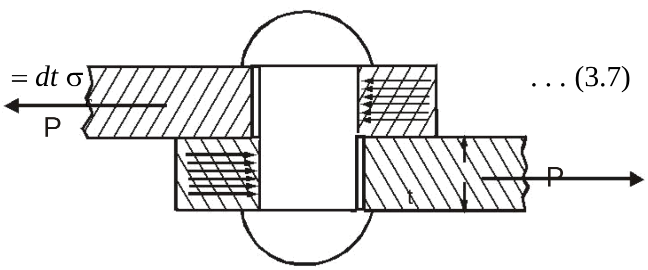


Figure 3.8 : Crushing of Rivet

The failure in this mode will not occur if

$$P_C \geq P \qquad \ldots (3.8)$$

where *P* is applied load per pitch length, and there is one rivet per pitch. If number of rivets is *n* in a pitch length then right hand side in Eq. (3.7) is multiplied by *n*.

Shearing of Plate Margin near the Rivet Hole

Figure 3.9 shows this mode of failure in which margin can shear along planes *ab* and *cd*.

If the length of margin is *m*, the area resisting this failure is,

$$A_{ms} = 2\ mt$$

If permissible shearing stress of plate is τ_s then load carrying capacity of the joint against shearing of the margin is,

$$P_{ms} = 2\ mt\ \tau_s \qquad \text{. . . (3.9)}$$

The failure in this case will not occur if

$$P_{ms} \geq P \qquad \text{. . . (3.10)}$$

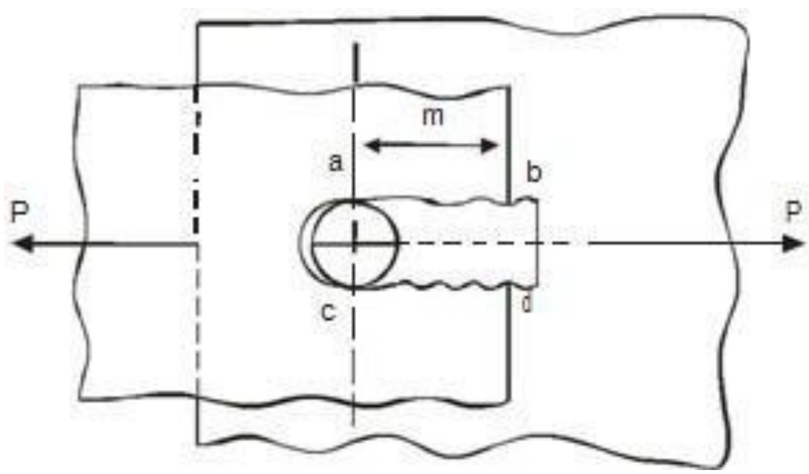


Figure 3.9: Shearing of Margin

The modes of failure discussed above are primary in nature and in certain cases they have to be considered uniquely. One such case is when rivets are arranged in lozenge form or diamond shape. This case will be discussed at proper stage.

In writing down the above equations for strength of the joint certain assumptions have been made. It is worthwhile to remember them. Most importantly it should be remembered that most direct stresses have been assumed to be induced in rivet and plate which may not be the case. However, ignorance of actual state of stress and its replacement by most direct stress is compensated by lowering the permissible values of stresses σ_t, τ_s and σ_c, i.e. by increasing factor of safety.

The assumptions made in calculations of strengths of joint in Eq. (3.1) through (3.10) are:

- The tensile load is equally distributed over pitch lengths.
- The load is equally distributed over all rivets.
- The bending of rivets does not occur.
- The rivet holes do not produce stress concentration. The plate at the hole is not weakened due to increase in diameter of the rivet during second head formation.
- The crushing pressure is uniformly distributed over the projected area of the rivet.
- Friction between contacting surfaces of plates is neglected.

Efficiency of Riveted Joints

If only a pitch length of solid or hole free plate is considered then its load carrying capacity will be,

$$P_1 = pt\,\sigma_t \qquad \ldots (3.11)$$

P_1 will apparently be greater then P_t, P_s, P_c or P_{ms}. The ratio of any of P_t, P_s, P_c or P_{ms} to P_1 is defined as the efficiency of the joint in that particular mode. Ideally P_t, P_s, P_c and P_{ms} all must be equal, but actually it may not be the case. The efficiency of the joint will be determined by least of P_t, P_s, P_c, and P_{ms}. Thus efficiency of the joint is,

$$\eta = \frac{\text{Least of } P_t, P_s, P_c \text{ and } P_{ms}}{P_t\,\sigma_t} \qquad \ldots (3.12)$$

The ideal that strengths in different modes of failure are equal is not achieved in a design because the rivet hole diameters and rivet diameters are standardized for technological convenience. Table 3.1 describes the average and maximum efficiencies of commercial boiler joints.

Table 3.1 : Efficiencies of Commercial Boiler Joints

Type of Joint	Average Efficiency %	Maximum Efficiency %
Lap Joints		
Single riveted	45-60	63.3
Double riveted	63-70	77.5
Triple riveted	72-80	86.5
Butt Joints		
Single riveted	55-60	63.3
Double riveted	70-83	86.6
Triple riveted	80-90	95.0
Quadruple riveted	85-94	98.1

Calculation of Hole Dia and Pitch

For an ideal joint the rivet should be equally strong against shearing and crushing.

Hence, from Eqs. (3.3) and (3.7), making $P_s = P_c$

$$\frac{\pi}{4} d^2 \tau_s = dt \sigma_c$$

$$\therefore \quad d = 1.274 \frac{\sigma_c}{\tau_c} t \quad \text{(in single shear)} \qquad \ldots (3.13)$$

If rivet is in double shear,

$$d = 0.637 \frac{\sigma_c}{\tau_s} t \qquad \ldots (3.14)$$

Generally τ_s = 60 MPa or N/mm^2

σ_c = 130 MPa or N/mm^2

giving $d = 2.75\, t$ in single shear . . . (3.15)

$d = 1.37\, t$ in double shear

Also equating right hand sides of Eqs. (3.1) and (3.3),

$$(p - d)\, t\, \sigma_t = \frac{\pi}{4} d^2 \tau_s$$

or

$$p = \frac{\pi d^2}{4t\, \sigma_t} \tau_s + d$$

Substituting τ_s = 60 MPa

σ_t = 75 MPa

$$p = 0.628 \frac{d^2}{t} + d$$

Using Eq. (3.15) in above equation

$$p = 2.73\, d \text{ (in single shear)}$$

$$p = 1.86\, d \text{ (in double shear)} \qquad \text{. . . (3.16)}$$

Equating right hand sides of Eqs. (3.7) and (3.9)

$$2mt\,\tau = dt\,\sigma_c$$

or

$$m = \frac{d}{2} \frac{\sigma_c}{\tau_s}$$

Substituting $\sigma_c = 130$ MPa

$\tau_s = 60$ MPa

$$m = 1.08\, d \qquad \text{. . . (3.17)}$$

There are several practical considerations due to which the design dimensions are modified. Most important of these is the pressure tightness of the joint, which is mainly achieved by caulking of the plate edges. The caulking becomes easier with short pitches and smaller rivets. It also makes it desirable that margin should be 1.5 *d* but not greater. The results in this section are indicative of calculation procedure and by no means be treated as standard formulae. These results are valid only for particular case and permissible stresses adopted. As a common practice for plate thickness greater than 8mm the diameter of rivet hole is determined by

$$d = 6\sqrt{t} \qquad \text{. . . (3.18)}$$

This is known as Unwin‟s formula.

It has been pointed out in the last sections that no attempt was made to derive formulae. The expressions for various load carrying capacities were written by examining the geometry. Therefore, you must see that in each problem the geometry is understood and then the expressions for forces are written. In the examples here we would see how we can approach to design a riveted joint.

Problem:

Design a double riveted lap joint for MS plates 9.5 mm thick. Calculate the efficiency of the joint. The permissible stresses are : σ_t = 90 MPa, τ_s = 75 MPa, σ_c = 150 MPa

Solution:

The joint to be designed is shown schematically in Figure 3.10.

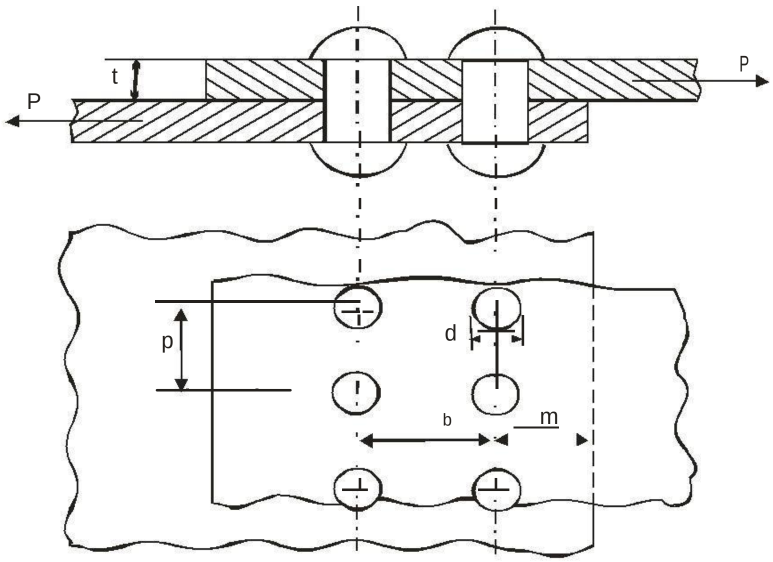


Figure 3.10

(a) **Dia. of Rivet Hole *d* :** It is determined by Unwin's formula, Eq. (3.18)

$$d = 6\sqrt{t}$$

or $$d = 6\sqrt{9.5} = 18.5 \text{ mm}$$

Pitch of the Joint, *p* : In a double riveted joint there are 4 rivets in a pitch length. The rivet diameter will be taken as diameter of the hole as difference between them is small. The rivets can fail in shear or due to crushing. We will first determine the shearing and crushing strength of a rivet and equate the smaller of two to the plate tearing strength to determine *p*.

Shearing strength of one rivet

$$= \frac{\pi}{4} d^2 \tau_s = \frac{\pi}{4} (18.5)^2 \, 75 = 20.16 \text{ kN} \qquad \ldots \text{(a)}$$

Crushing strength of one rivet

$$= \sigma_c \, dt = 150 \times 18.5 \times 9.5 = 26.36 \text{ kN} \qquad \ldots \text{(b)}$$

From (a) and (b) it is seen that the rivet is weaker in shear.

$\therefore$ We will equate tearing strength of plate with shearing strength of rivets in a pitch length.

There are two rivets in the pitch length.

$$\therefore \qquad \sigma_t (p - d) t = 2 \times \frac{\pi}{4} d^2 \tau_s$$

$$\text{or} \qquad p = \frac{\pi d^2}{2t} \frac{\tau_s}{\sigma_t} + d = \frac{\pi}{2} \frac{(18.5)^2}{9.5} \frac{75}{90} + 18.5$$

$$\text{or} \qquad p = 65.55 \text{ mm say } 65.7 \text{ mm} \qquad \ldots \text{(ii)}$$

The pitch should be such that head forming operation is not hindered. The practice dictates that $p \geq 3\,d$ so that head forming is permitted.

$3\,d = 55.5$ mm, and hence the value of p obtained in (ii) is acceptable.

The back Pitch p_b : It must be between 2.5 d to 3.0 d. For chain riveting the higher value is preferred for the reason of head forming

$$p_b = 3d = 3 \times 18.5 = 55.5 \text{ mm} \qquad \ldots \text{(iii)}$$

Margin, *m* : m is determined by equating shearing strength of rivet (smaller of shearing and crushing strengths of rivet). Remember that there are two rivets per pitch length :

$$\therefore \qquad 2mt\,\tau_s = 2 \frac{\pi}{4} d^2 \tau_s$$

$$\therefore \qquad m = \frac{\pi}{4} \frac{d^2}{t} = \frac{\pi}{4} \frac{(18.5)^2}{9.5} = 28.3 \text{ mm} \qquad \ldots \text{(iv)}$$

The minimum acceptable value of m is 1.5 d = 27.5 mm hence

m = 28.3 mm is acceptable.

Thus the design is completed with

d = 18.5 mm, p = 65.7 mm, p_b = 55.5 mm, m = 28.3 mm

The diameter is standardized, apparently based on drill size. Normally fractions like 18.5 mm may not be accepted. The rivet diameters are less than hole diameter by 1 mm. Yet the head formation process increases rivet diameter. We are not yet describing standard hole and rivet diameters.

We postpone it for the time being.

Efficiency of Joint

Tensile strength of plate without holes, per pitch length

$$P = \sigma_t pt = 90 \times 65.7 \times 9.5 = 56.2 \text{ kN} \quad \ldots (c)$$

Shearing strength of rivets in a pitch length

$$P_s = 2 \times \tau_s \times \frac{\pi}{4} d^2 = 2 \times 75 \times \frac{\pi}{4} (18.5)^2 = 40.3 \text{ kN} \quad \ldots (d)$$

Crushing strength of rivets in a pitch length

$$P_c = 2 \times \sigma_c \times d \times t = 2 \times 150 \times 18.5 \times 9.5 = 52.7 \text{ kN} \ldots\ldots (e)$$

The tearing strength of plate with one hole in a pitch length

$$P_t = \sigma_t (p - d) t = 90 (65.7 - 18.5) 9.5 = 40.36 \text{ kN} \ldots\ldots (f)$$

The shearing strength of margin

$$P_{ms} = 2 \tau_s mt = 2 \times 75 \times 28.3 \times 9.5 = 40.32 \text{ kN} \ldots\ldots (g)$$

Out of all P_s, P_c, P_t and P_{ms}, the lowest is P_m

$$\eta = \frac{P_s}{P_1} = \frac{40.3}{56.2} = 71.7\% \ldots\ldots (h)$$

The design values are

d = 18.5 mm, p = 65.7 mm, p_b = 55.5 mm, m = 28.3 mm, η = 71.7%

Riveted Joints in Structures

For trusses, bridges or girders, etc. where the width of the plates is known in advance lozenge type or diamond shaped joints are preferred. These joints have uniform or equal strengths in all modes of failure. Marginal adjustments in calculated dimensions may slightly reduce or increase strength in any particular mode. The joints are usually of double cover butt type with rivets so arranged that there is only one rivet in the outermost row and their number increases towards inner row. (See Figure 3.11). Since the plate width is known in advance, its strength in tension can be determined. Thus the load carrying capacity of the joint

$$P = \sigma \quad (b - d)\, t$$

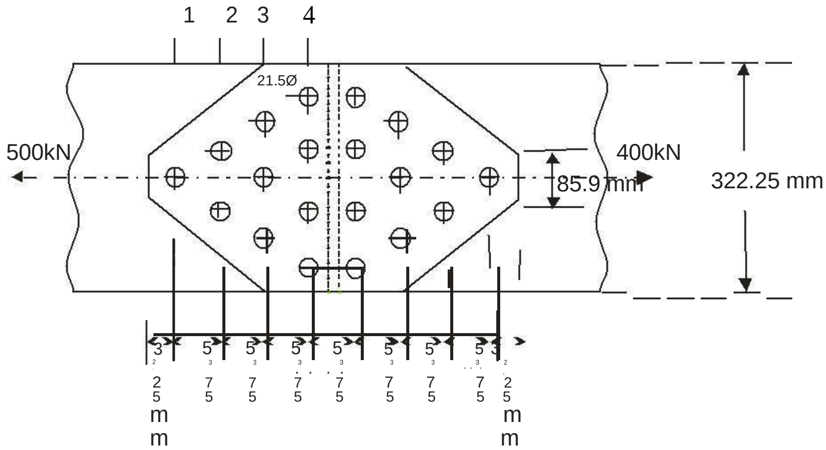


Figure 3.11

Here b is the plate width; t is thickness and d, the diameter of the hole. σ_t is the permissible tensile stress. The rivet diameter is determined by Unwin‟s relationship. The determination of number of rivets is the main task for the required force that is carried by the member. Of course we would first determine whether the strength of rivet is less in shear or in crushing which would depend upon relative magnitudes of τ_s and σ_c as well as on the cross sectional area of the rivet and its projected area. The next step would be to arrange the rivets in diamond shape as shown in Figure 3.11. Then we decide upon the pitch, back pitch and margin.

The joint is designed not to tear in the outer most rows, i.e. row 1 in Figure 3.11. Then the row 2, which is the next inner row and weakened by two rivet holes, is subjected to tearing. Note that this tearing is possible if rivet in the outermost row (or row 1) shears or is crushed at the same time. That means this type of joint has one more possible mode of failure which comprises tearing along an inner row accompanied by shearing or crushing of rivets in all outer rows. The strength in this mode is denoted by P_{ts} or P_{tc} and one more suffix may be used to denote the row in which tearing will occur.

Thus $$P_{ts2} = \sigma_t (b - 2d)\, t + \tau_s \frac{\pi}{4} d^2$$

or $$P_{tc2} = \sigma_t (b - 2d)\, t + \sigma_c d t$$

And $$P_{ts3} = \sigma_t (b - 3d)\, t + 3\tau_s \frac{\pi}{4} d^2$$

or $$P_{tc3} = \sigma_t (b - 3d)\, t + 3\sigma_c dt$$

Problem:

Two steel plates 12.5 mm thick are required to carry a tensile load of 500 kN in a double cover butt joint. Calculate the width of the plate if it is not to be weakened by more than one rivet hole. Design the butt joint completely and show dimensions on a sketch. The ultimate values of strengths are as follow

Plates in tension	–	600 MPa
Rivet in Shear	–	490 MPa
Plate and rivet in crushing	–	920 MPa

Use a factor of safety of 4.5.

Also use following standards :

Rivet Holes

From 13.5 mm to 25.5 mm in steps of 2 mm and from 27 mm to 42 mm in steps of 3 mm.

Rivets

1.5 mm less than rivet hole diameter upto 24 mm in steps of 2 mm and
2 mm less than rivet hole diameter from 25 mm to 39 mm in steps of 3 mm.

Solution:

The permissible stresses are:

$$\sigma_t = \frac{600}{4.5} = 133 \text{ N/mm}^2$$

$$\tau_s = \frac{490}{4.5} = 109 \text{ N/mm}^2$$

$$\sigma_c = \frac{920}{4.5} = 204 \text{ N/mm}^2$$

Diameter of the Rivet Hole : Use „Unwin"s Formula

$$d = 6\sqrt{t} = 6\sqrt{12.5} = 21.21 \text{ mm}$$

From standards, $d = 21.5$ mm

Hence the rivet dia, $d_1 = 20$ mm . . . (i)

Compare shear strength and crushing strength of one rivet.

$$P_s = \tau_s \frac{\pi}{4} d_1^2 = 109 \times \frac{\pi}{4} \times (20)^2 = 34 \text{ kN}$$

$$P_c = \sigma_c d_1 t = 204 \times 20 \times 12.5 = 51 \text{ kN}$$

Thus rivet in crushing is stronger than in shear.

Width of the Plate

Consider the tensile strength of the weakest section of the plate, i.e. the row which is weakened by one rivet hole.

$$P = \sigma_t (b - d)\, t$$

or $$500 \times 10^3 = 133\,(b - 21.5)\,12.5 \quad \ldots \text{(ii)}$$

$$b = 300.75 + 21.5 = 322.25 \text{ mm}$$

Number of Rivets

The rivets are in double shear in double cover butt joint. The strength in double shear is the strength in single shear. Also we assume that the head formation does not change rivet diameter.

$$n\,P_s = P \text{ i.e. } n\, 59.5 = 500 \text{ kN}$$

Also crushing strength

$$n\,P_c = P \text{ i.e. } n \times 51 = 500$$

or $$n = \frac{500}{51} = 9.8 \text{ say } 10 \quad \ldots \text{(iii)}$$

We will see that 10 rivets are better arranged.

Rivet Arrangement

Ten rivet can be easily arranged in four rows : 1, 2, 3 and 4 can be arranged in rows 1, 2, 3, and 4 which will be a good arrangement. We should ensure that 10 rivets should not weaken the plate. The arrangement is shown in Figure 3.11. The pitch of the rivets is determined by geometric consideration. The innermost or 4th row should have a margin of

$1.5\,d = 1.5 \times 21.5 = 32.25$ mm from the edge.

Thus, the distance between centers of two extreme rivets in row

4 $b - 2\,m = 322.25 - \times 32.25 = 257.75$ mm

Obviously this distance is equal to 3 $p = 257.75 = p\,85.9$ mm $\quad \ldots$ (iv)

The distance between the rows, p_b should be between 2.5 to 3 d

$p_b = 2.5 \times 21.5 = 53.75$ mm

Cover Plate

Theoretically the cover plate may have thickness of $t/2$ but practically the thickness $t_c = 0.62\ t$

$$t_C = 0.625 \times 12.5 = 7.8 \text{ mm.} \quad . . . \text{(vi)}$$

The cover plates are given the diamond shape so as to accommodate all the rivets (See Figure 3.11).

Efficiency

Shearing strength of 10 rivets is double shear

$$P_s = 1.75 \times 10 \times \frac{\pi}{4} d_1^2 \tau_s$$

$$= 1.75 \times 10 \times \frac{\pi}{4} (20)^2 \times 109 = 17.5 \times 34 = 595 \text{ kN} \qquad \ldots \text{(a)}$$

Crushing strength of 10 rivets

$$P_c = 10 \times \sigma_c d_1 t = 10 \times 51 = 510 \text{ kN} \qquad \ldots \text{(b)}$$

Tearing strength of plate along weakest section, i.e. along row 1

$$P_{t1} = (b - d) t \sigma_t$$

$$= 133 (322.25 - 21.5) 12.5 = 500 \text{ kN} \qquad \ldots \text{(c)}$$

Strength for tearing along second row and crushing of one rivet in row 1

$$P_{tc2} = 133 (322.25 - 2 \times 21.5) 12.5 + \sigma_c d_1 t$$

$$= 133 \times 279.25 \times 12.5 + 204 \times 20 \times 12.5$$

$$= 464.25 + 51 \text{ kN}$$

or $P_{tc2} = 515.5$ kN $\qquad \ldots$ (d)

Strength for tearing along third row and crushing of 3 rivets in row 1 and 2

$$P_{tc3} = 133 (322.25 - 3 \times 21.5) 12.5 + \sigma_c \times 3 d_1 t$$

$$= 133 \times 257.75 \times 12.5 + 3 \times 51 \times 10^3 = 428.5 + 3 \times 51 \text{ kN} \qquad \ldots \text{(e)}$$

or $P_{tc3} = 581.5$ kN

Strength for tearing along fourth row and crushing of rivets in other rows

$$P_{tc4} = 133 (322.25 - 4 \times 21.5) 12.5 + 6 \times 51 \times 10^3$$

or $P_{tc4} = 699$ kN $\qquad \ldots$ (f)

(e) and (f) were expected to be more than (d), yet the calculations have been made for the sake of completeness.

Strength of solid plate without hole

$$P_t = \sigma_t bt = 133 \times 322.25 \times 12.5 = 535.74 \text{ kN} \qquad \ldots \text{(g)}$$

The least of all strengths from (a) through (f) is P_{t1} = 500 kN

$\eta = P_{tl} / P_t$ = (500/535.74) 100 = 93.3%

Joints for Boilers and Pressure Vessels

The boiler and pressure vessels are cylindrical in shape and withstand internal pressure. The vessels are required to be leak proof. The maintenance of pressure and safety of boilers have prompted several standards. ASME boiler code, Board of Trade (BOT) Rules, Indian Boiler Regulations (IBR) and ISI standards are available for design of boilers and pressure vessels.

The cylindrical pressure vessel is identified by two dimensions, viz., the length and diameter. The cylinders are made from plates and whole length may not be obtained from single sheet hence cylindrical sections are obtained by bending sheets and joining edges by riveted joint. The sections are then joined together by another riveted joint along circumference. Thus there are two types of joint longitudinal and circumferential (see Figure 3.12). The longitudinal joint will bear hoop stress (σ_h) and circumferential joint bears longitudinal stress (σ_l). Since $\sigma_h = 2\sigma_l$, the longitudinal joint will have to be two times as strong as circumferential joint. Therefore, longitudinal joints are always made butt joints whereas the circumferential joints are mode as lap joints.

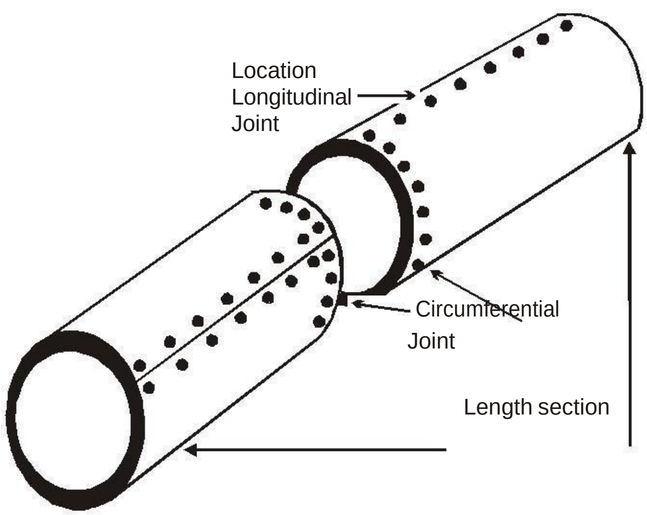


Figure 3.12 : Longitudinal and Circumferential Location for Riveted Joints

The steps followed in design of boiler riveted joints are same as followed in any joint design. They are mentioned here as described in IBR.

Design Procedure for Longitudinal Butt Joint

Determine Thickness of Boiler Shell (*t*)

The efficiency of the joint is chosen from Table 3.1 and for pressure σ_r, inner diameter *D* and permissible tensile stress σ_t, the thickness is calculated from,

$$t = \frac{\sigma_r D}{2\sigma_t \eta} + 1 \text{ mm} \qquad \ldots (3.19)$$

The diameter and thickness will further guide in respect of rivet arrangement. Table 3.2 can be used for this purpose.

Table 3.2 : Suggested Rivet Arrangement

Dia. of Shell (mm)	**Thickness of Shell (mm)**	**Rivet Arrangement**
610-1830	6-12.5	Double riveted
915-2130	8-25.0	Triple riveted
1525-2740	9.5-31.75	Quadruple riveted

Determine Rivet Hole Diameter (*d*) and Rivet Diameter (d_1)

Unwin‟s formula, giving $d = 6\sqrt{t}$ is used if $t \geq 8$ mm. In very rare case if $t < 8$ mm, *d* is calculated by equating shearing strength and crushing strength of rivet. The diameter of hole must be rounded off to the nearest standard value with the help of Table 3.3, and the diameter of rivet also established.

Table 3.3 : Standard Rivet Hole and Rivet Diameters

d **(mm)**	13	15	17	19	21	23	25	28.5	31.5	34.5	37.5	41	44
d_1 (mm)	12	14	16	18	20	22	24	27	30	33	36	39	42

Determine Pitch of the Rivet (*p*)

The minimum pitch is 2 *d* to accommodate the dies to form head. The pitch is calculated by equating tearing strength with shearing or crushing strength of rivet(s). However, the pitch should not exceed certain value for leak proof nature of the joint. The maximum value of *p* is given by following equation.

$$p_{max} = C \times t + 41.28 \text{ mm} \qquad \ldots (3.20)$$

The value of C is given in Table 3.4. If by calculation p turns out to be less than p_{max}, it will be acceptable.

Table 3.4 : The Value of Constant for Maximum Pitch

Number of Rivets	Lap Joint	Butt Joint Single Cover	Butt Joint Double Cover
1	1.31	1.53	1.75
2	2.62	3.06	3.50
3	3.47	4.05	4.63
4	4.17	–	5.52
5	–	–	6.00

Determine Back Pitch (p_b)

(a) For both lap and butt joints having equal number of rivets in different rows p_b is given as

$$p_b = (0.33\,p + 0.67\,d) \text{ mm for zig-zag} \qquad \ldots (3.21)$$

and

$$p_b = 2\,d \qquad \ldots (3.22)$$

(b) For joints in which number of rivets in outer rows is half of that in inner rows which are chain riveted p_b should be greater of the values calculated from Eqs. (3.21) and (3.22). However, the value of p_b for rows having full number of rivets will not be less than $2\,d$.

(c) The third case arises for joints having inner rows zig-zag riveted and outer rows having half the number of rivets as inner rows where

$$p_b = (0.2\,p + 1.15\,d) \text{ mm} \qquad \ldots (3.23)$$

The back pitch for zig-zag riveted inner rows will be

$$p_b = (0.165\,p + 0.67\,d) \text{ mm} \qquad \ldots (3.24)$$

The pitch p in above equations is the one in outer row, i.e. away from butting edges.

Determine Thickness of the Cover Plate (t_c)

(a) For single butt cover with chain riveting

$$t_c = 1.125\,t \qquad \ldots (3.25)$$

(b) For single cover with pitch in the outer row being twice that in the inner row

$$t_c = 1.125 \left(\frac{p - d}{p - 2d} \right) t \qquad \ldots (3.26)$$

(c) For double cover of equal width and chain riveting

$$t_C = 0.625\, t \qquad \ldots (3.27)$$

(d) For double cover of equal width with pitch in the outer row being twice that in the inner row

$$t_c = \left(\frac{p - d}{p - 2d}\right) t \qquad \ldots (3.28)$$

(e) For double cover of unequal width (wider cover on the inside)

$$\left.\begin{aligned} t_{c1} &= 0.75\, t \quad \text{(for cover on the inside)} \\ t_{c2} &= 0.625\, t \quad \text{(for cover on the outside)} \end{aligned}\right\} \qquad \ldots (3.29)$$

Determine margin, $m = 1.5\, d$ . . . (3.30)

Determine Caulking Pitch, p_t

The pitch of rivets in the row nearest to the edge must be as small as possible to avoid leakage. This pitch is called caulking pitch and helps edges to be caulked effectively (see Figure 3.3). A rough rule is that this pitch should not be greater than S_{tc}. The caulking pitch is, however, calculated from following :

$$p_c = d + 13.8 \frac{t^{3/4}}{(\sigma_r)^{1/4}} \qquad \ldots (3.31)$$

This is an empirical relation in which σ_r the pressure is used in N/mm^2.

Design Procedure for Circumferential Lap Joint

The thickness of the shell, the diameter of the rivet hole, back pitch and margin are calculated in the same way as for longitudinal butt joint. The other quantities are presented under.

Number of Rivets (n)

The rivets are in single shear and all of them are subjected to shear when pressure, σ_r acting on the circular section of the cylindrical space tends to separate two length sections of the vessel.

$$n\, \tau_s \frac{\pi}{4} d_1^2 = \sigma_r \frac{\pi}{4} D^2$$

$$n = \frac{\sigma_r D^2}{\tau_s d_1^2} \qquad \ldots (3.32)$$

$_s d_1$

Pitch, (*p*)

Efficiency of the lap joint η can be taken as half of the efficiency of the longitudinal butt joint. The efficiency of the lap joint is calculated on the basis of tearing load capacity of the joint which turns out to be least of strengths in all modes.

Thus, $$\eta = \frac{p - d}{p} \qquad \ldots (3.33)$$

Number of Rows, (*N*)

The rivets are placed all along the circumferences of the shell. Hence number of rivets in one row.

$$n_1 = \frac{\pi (D + t)}{p}$$

Hence total number of rivets in $N\, n_1 = n$.

$$N = \frac{n\, p}{\pi (D + t)} \qquad \ldots (3.34)$$

Whether the joint will be single riveted or multiple riveted will be decided by *N*. If *N* turns out to be less than 1, a single riveted joint will serve the purpose. In any case the pitch will have to satisfy the condition of caulking.

Overlap of Shell Length Section (*l*)

$$l = (N - 1)\, p_b + 2m \qquad \ldots (3.35)$$

Problem #:

Inner diameter of a boiler is 1500 mm and the steam pressure is 2 N/mm^2. Use a proper joint along the length and design it completely. Use following permissible values of stress.

Tension σ_t = 90 MPa Shear τ_s = 75 MPa Crushing σ_c = 150 MPa

Solution

Thickness of the Shell (*t*)

From Table 3.2 for shell diameter of 1500 mm a double riveted butt joint is recommended and from Table 3.1 we can use an efficiency of 80%.

From Eq. (3.17).

$$\therefore = \frac{\sigma_r D}{2\sigma_t \eta} + 1 \text{ mm}$$

$$= \frac{2 \times 1500}{2 \times 90 \times 0.8} + 1 = 20.8 + 1$$

or $t = 21.8$ mm say 22 mm

Rivet Hole Diameter (*d*)

From Eq. (3.18)

$$d = 6\sqrt{t} = 6\sqrt{22} = 28.14 \text{ mm}$$

The nearest standard value of hole diameter is 28.5 mm, and corresponding rivet diameter is 27 mm. $d_1 = 27$ mm.

Pitch (*p*)

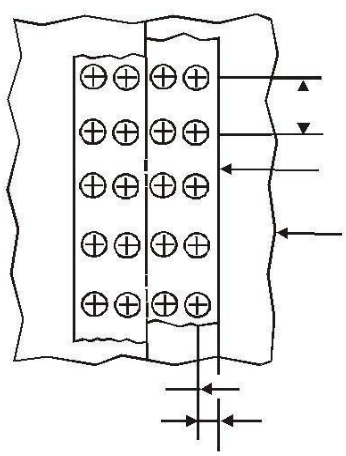
In one pitch length there are two rivets which may shear or crush (Figure 3.13).

The shear strength of one rivet in double shear

$$P_{s1} = 1.75 \times \frac{\pi}{4} d_1^2 \tau_s = 1.75 \times \frac{\pi}{4} (27)^2 \times 75 = 75.2 \text{ kN}$$

The crushing strength of one rivet

$P = t\, d\, \sigma = 22 \times 27 \times 150 = 89.1$ kN



The rivet is weaker in shearing. Equating tearing strength of plate with shearing strength of 2 rivets in a pitch length,

$$(p - d)\, t\, \sigma_t = 2 \times 1.75 \times \frac{\pi}{4} d_1^2 \tau_s$$

$$p = 3.5 \times \frac{\pi}{4} (27)^2 \times \frac{75}{22 \times 90} + 28.5 = 104.4 \text{mm}$$

Check for maximum value of pitch from Eq. (3.20). From Table 3.4 for 2 rivets in a pitch length for a double cover double riveted joint the value of $C = 3.5$.

$$\therefore\ P_{max} = C \times t + 41.28 = 3.5 \times 22 + 41.28 = 118.28 \text{ mm}$$

The min. pitch is 2 d. Hence calculated value of p = 104.4 mm is acceptable.

We may choose p = 105 mm.

Back Pitch (p_b)

$p_b = 0.33\ p + 0.67\ d = 0.33 \times 105 + 0.67 \times 28.5\ p_b = 34.65 + 19.1 = 53.75$ mm

However, p_b should not be less than 2 d or 57 mm p_b = 57 mm.

Thickness of Cover Plate (t_c)

The joint has two equal cover plates. From Eq. (3.27) $t_c = 0.625\ t = 13.75$ mm

Margin (m)

$m = 1.5\ d = 42.75$ mm

Efficiency (η)

The shearing strength of the joint

$P_S = 2 \times 75.2 = 150.4$ kN

The crushing Strength of the joint

$P_c = 2 \times 89.1 = 178.2$ kN

The tearing strength of plate with holes

$$P_t = (p - d)\ t\ \sigma_t = (105 - 28.5)\ 22 \times 90$$

or $\quad P_t = 151.47$ kN

The tensile strength of plate without holes

$P = pt\ \sigma = 105 \times 22 \times 90 = 208$ kN

P_S is least of P_S, P_C and P_t

$$\therefore \quad \eta = \frac{P_S}{P} = \frac{150.4}{208} = 72.3\%$$

Problem:

Design a circumferential lap joint for boiler shell of Problem #.

Solution

The thickness of the shell t and rivet hole diameter d (and rivet diameter d_1) will remain same, i.e.

t = 22 mm, d = 28.5 mm, d_1 = 27 mm Number

of rivets (n) : Use Eq. (2.32)

$$n = \frac{\sigma_r D^2}{\tau_S d_1^2} \qquad \sigma_r \text{ is pressure in boiler}$$

$$n = \frac{2}{75}\left(\frac{1500}{27}\right)^2 = 82.3 \text{ say } 83$$

Pitch (*p*)

These rivets (83 in number) have to be placed along circumference and preferably in two rows for better leak proofing. However, arranging rivets in two rows will alter the number of rivets. We will first determine pitch from efficiency, making efficiency in plate tearing mode as the least. At best it is required that efficiency of the circumferential joint should be 50% of the efficiency of the longitudinal joint. So in the case

$$\eta = \frac{0.8}{2} = 0.4$$

$$\eta = \frac{p - d}{p} = 0.4 \text{ so that } p - 0.4\,p = d$$

or $$p = \frac{d}{0.6} = \frac{28.5}{0.6} = 47.5 \text{ mm}$$

Number of Rows (*N*)

Apparently n number of rivets are to be distributed with p = 47.5 mm around a circumference of $\pi\,(D + t)$

$$\therefore \quad N = \frac{n\,p}{\pi\,(1500 + 28.5)} = \frac{83 + 47.5}{\pi \times 1528.5} = 0.82$$

To make N = 2 and keeping p = 47.5 mm, n will increase,

$$2 = \frac{n \times 47.5}{\pi \times 1528.5} \text{ giving } n = 202$$

Now choosing n = 202 will alter p

$$p = \frac{2 \times \pi \times 1528.5}{202} = 47.54 \text{ mm}$$

With number of rivets 2 per pitch length the constant C from Table 3.4 is 3.06 and using Eq. (3.20)

$$p_{max} = Ct + 41.28 = 3.06 \times 22 + 41.28 = 108.6 \text{ mm}$$

But for convenience of caulking p should be at least 2 d.

$\therefore \quad p = 2 \times 28.5 = 57$ mm

This will further alter number of rivets as

$$n = \frac{2\pi \times 1528.8}{57} \quad 168.5 \text{ say } 168$$

So that $$p = \frac{2\pi \times 1528.8.5}{168} = 57.16 \text{ mm}$$

$\therefore$ $n = 168$, $p = 57.16$ mm, $N = 2$

$$\eta = \frac{p - d}{p} = \frac{57.16 - 28.5}{57.16} = 50.44\%$$

Back Pitch (P_b)

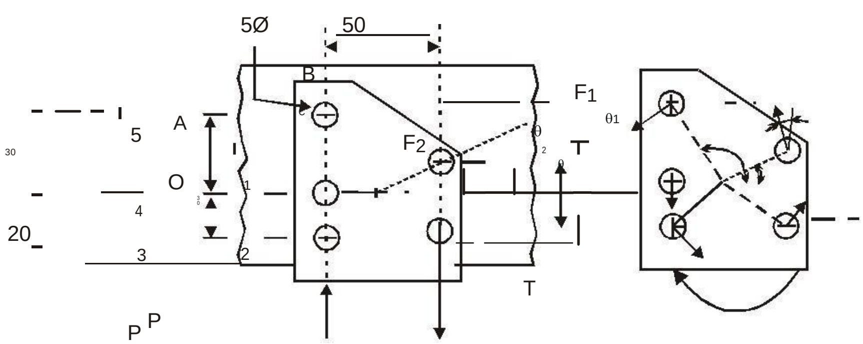
The rivets can be arranged in zig-zag rows so that $p_b = 2\,d = 57$ mm

Margin (m) : $m = 1.5\,d = 1.5 \times 28.5 = 42.75$ mm Overlap (l) : $l = (N - 1)\,p_b + 2m$

or $l = 57.0 + 2 \times 42.75 = 142.5$ mm.

Torsional Loading and Eccentric Loading of Riveted Joint

Plate *A* is riveted to structural element *B*. A torque is applied to the Plate *A*. The plate will rotate, of course by slight elastic amount, about some point as *o* in Figure 3.15(a). It is not wrong to assume that any straight line such as *oc* which passes through the centre of a rivet remains straight before and after application of the torque. Then the deformation, hence strain and so the average shearing stress across the section of the rivet will be proportional to the distance between *o* and the centre of the rivet. Since the average shearing stress is equal to the shearing force divided by area of cross section of the rivet, the shearing force on the rivet will be proportional to the distance between *o* and centre of the rivet. The direction of this force will be perpendicular to the joining line.



(a) (b)

Figure 3.15

The forces F_1, F_2, etc. on individual rivets are shown in Figure 3.15(b). For satisfying condition of equilibrium, components of forces in vertical direction should sum up to zero. If the forces F_1, F_2, etc. make angles θ_1, θ_2, etc. respectively with y-axis, then

$$F_1 \cos\theta_1 + F_2 \cos\theta_2 + \ldots + F_n \cos\theta_n = 0$$

or $$\sum_{i=1}^{n} F_i \cos\theta_i = 0 \qquad \ldots \text{(i)}$$

But $$F_i = \tau_i A_i = \frac{\tau_i}{r_i} r_i A_i$$

And since $$\tau_i \,\alpha\, r_i \quad \text{or } \tau_i = k\, r_i$$

$\therefore$ $$F_i = k r_i A_i \qquad \ldots \text{(ii)}$$

Here k is a constant, τ_i = shearing stress in i^{th} rivet whose area of cross section is A_i and its centre is at a distance r_i from o.

Use (ii) and (i) to obtain

$$k \sum_{1}^{n} r_i A_i \cos\theta_i = 0$$

See from Figure 2.15(b) that $r_i \cos\theta_i = x$

$\therefore$ $$k \sum_{1}^{n} x A_i = 0$$

Which is same as $k \bar{x} A_t = 0$.

Where $\bar{x}$ is the x-coordinate of centroid of all the rivet and sum of their areas of cross sections is A_t. And since neither k nor A_t is zero therefore, $\bar{x} = 0$. If then we consider sum of forces along x-axis we would arrive at the result $\bar{y} = 0$. This means that o is the point coinciding with the centroid of the rivet area system

Problem:

In Figure 3.15(a) the distances between columns and rows of rivets are shown. Each rivet is 5 mm in diameter and force P = 1 kN. Calculate the maximum shearing stress in rivet.

Solution

The five rivets have been numbered as 1, 2, . . . , 5. Take centre of rivet 3 as origin and x and y axes along 32 and 35 respectively. Areas of all rivets is

$A = \frac{\pi}{4}(5)^2 = 19.64\ \text{mm}^2$. If $\bar{x}$ and $\bar{y}$ are the coordinates of the centroid, then

$$50\,A + 50\,A = 5A\,\bar{x}$$

Hence, $\bar{x} = 20$ mm

Also, $50\,A + 20\,A + 30\,A = 5\,A\,\bar{y}\,, \quad \bar{y} = 20$ mm

Hence centroid is on the horizontal line through rivet 4. We can calculate various distances of rivet centres from centroid.

$$r_1 = \sqrt{10^2 + 30^2} = 10\sqrt{10}$$

$$r_2 = \sqrt{20^2 + 30^2} = 10\sqrt{13}$$

$$r_3 = \sqrt{20^2 + 20^2} = 10\sqrt{8}$$

$$r_4 = \sqrt{20^2 + 0} = 10 \times 2$$

$$r_5 = \sqrt{30^2 + 20^2} = 10\sqrt{13}$$

Now $F_1 = k\,r_1 = k\,10\sqrt{10}$, moment of F_1 about O,

$$M_1 = k\,r_1^2 = 1000\,k$$

$$F_2 = k\,r_2 = k\,10\sqrt{13},\; M_2 = 1300\,k$$

$$\frac{F_1}{F_2} = \frac{r_1}{r_2} = \frac{\sqrt{10}}{13} \quad \text{or } F_1 = \sqrt{\frac{10}{13}}\,F_2$$

We can find each of F_1, F_2, F_3, F_4 and F_5 in terms of k or we can find each force in terms of F_2. We may like to choose F_2 because F_2 is grater than all other forces because r_2 is larger than all other r.

$$F_3 = \sqrt{\frac{8}{13}}\,F_2\,,\; F_4 = \frac{2}{\sqrt{13}}\,F_2\,,\; F_5 = F_2$$

Taking moments of all forces about O and equating with the applied moment of 50 P = 50000 N mm.

$$\sqrt{\frac{10}{13}}\,F_2\,10\sqrt{10} + F_2\,10\sqrt{13} + \sqrt{\frac{8}{13}}\,F_2\,10\sqrt{8} + \frac{2}{\sqrt{13}}\,F_2\,10 \times 2 + F_2\,10\sqrt{13} = 5 \times 10^4$$

$$\frac{100\,F_2}{\sqrt{13}} + \frac{130\,F_2}{\sqrt{13}} + \frac{80\,F_2}{\sqrt{13}} + \frac{40\,F_2}{\sqrt{13}} + \frac{130\,F_2}{\sqrt{13}} = 5 \times 10^4$$

$$\therefore \quad F_2 = \frac{500\sqrt{13}}{}$$

4.8 = 375.6 N

$\therefore$ Maximum shearing stress = $\dfrac{F_2}{A} = \dfrac{375.6}{19.64} = 19.12$ N/mm^2

Problem:

Figure 3.16(a) shows a plate riveted on to a vertical column with three rivets placed at three corners of an equilateral triangle of size 75 mm. A load of 37 kN acts on the plate at a distance of 125 mm from vertical line through a rivet as shown in Figure 3.16(a). If the permissible stress in rivet is 60 N/mm^2 calculate the diameter of the rivet.

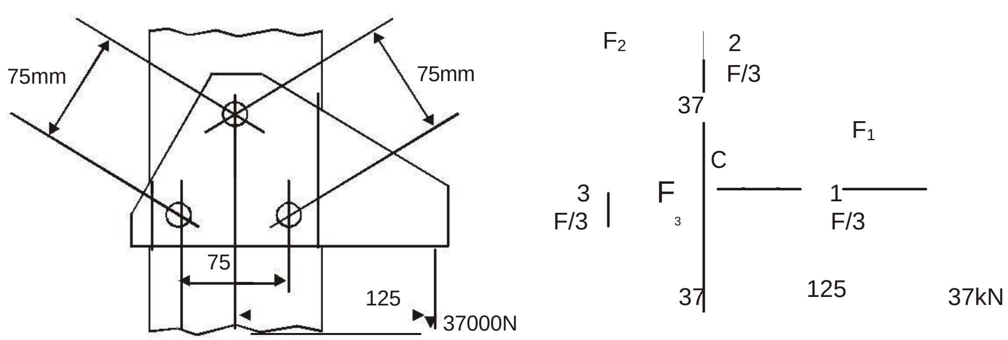
The rivets are at corners of equilateral triangle hence their centroid will be at the centroid of the triangle, *C*. Each of rivets 1, 2 and 3 will be at the same distance from *C*.

$$r_1 = r_2 = r_3 = \frac{2}{3}\sqrt{75^2 - 37.5^2} = \frac{2}{3}\sqrt{5625 - 1406.25} = 43.3 \text{ mm}$$

The force of 37 kN is acting at a distance of 125 mm from vertical line through the centroid as shown in Figure 3.16(b). Apply two forces, each equal to 37 kN but in opposite direction at *C*. Combining two 37 kN forces as shown in the figure, we are left with a couple 37 kN by 125 mm and a vertical force 37 kN acting downward. This force will be distributed equally on three rivets, i.e. if F = 37 kN, then $\frac{F}{3}$ will be a direct shearing force acting downward on each rivet as shown in Figure 3.16(b). The moment of the couple, $T = 37 \times 125 = 4625$ kN mm will be balanced by moments of forces F_1, F_2 and F_3 about C. F_1, F_2 and F_3 are perpendicular to r_1, r_2 and r_3, respectively and each is proportional to its r. incidentally due to symmetry of equilateral triangle r_i, r_2 and r_3 are mutually equal and hence $F_1 = F_2 = F_3$.

Equate moments $F_1 r_1 + F_2 r_2 + F_3 r_3 = T$

$3F_1 r_1 = T$



(a) (b)

Figure 3.16

Or by putting r_1 = 43.3 mm

$$F_1 = \frac{4625}{3 \times 43.3} = 35.6 \text{ kN}$$

$\therefore$ Resultant force on rivet 3

$$R = \sqrt{\left(\frac{F}{3}\right)^2 + F_3^2 + 2\,\frac{F}{3}\,F_3 \cos\theta}$$

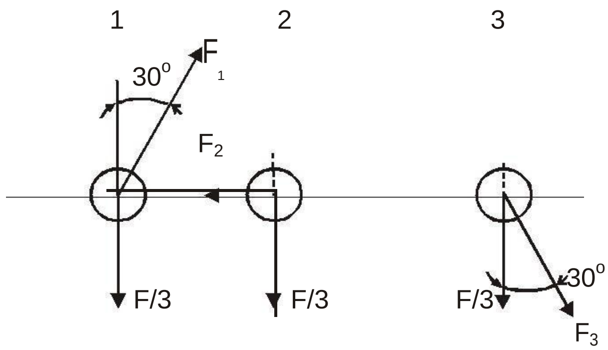


Figure 3.17

Put F = 37 kN, F_3 = 35.6 kN, $\theta = 30^o$

$$R = \sqrt{(12.33)^2 + (35.6)^2 + 2 \times 12.33 \times (35.6) \cos 30}$$

$$= \sqrt{152 + 1267.36 + 761} = \sqrt{2180.36} = 46.7 \text{ kN}$$ With τ as permissible shearing stress and d as diameter of rivet

$$R = \frac{\pi}{4} d^2 \tau$$

Use $R = 46.7 \times 10^3$ N, $\tau = 60$ N/m^{2}m

$$d^2 = \frac{4 \times 46.7}{\pi \; 60} \times 10^3 = 991$$

$\therefore$ d = 31.5 mm

We had assumed that all rivets have same diameter that is how we had determined centroid. Hence all three rivets will have the diameter of 31.5 mm each.

Welded Joints

Introduction

The problem of connecting plates was first solved through riveted connections but the development that occurred during World War-II saw the welded joints replace riveted joints in most applications. The ship building industry was perhaps in the fore front and large ships in excess of 10,000 in number were build with welded structures welding technology, indeed provided several advantages. The ease of processing and weight reduction were the identifiable advantages in the beginning. The automation and variety of welding processes have now become the most obvious advantages the technological developments have included several steels and even non-ferrous metals in the lists of weldable materials.

Welded Connections

Welding is a process of joining two or more pieces of metals. The process is of course adopted to obtain specific shapes and sizes to perform specific function. In the process of welding the temperature of metal to be jointed is raised to a level so that the metal becomes plastic or fluid. When metal is just plastic then pieces to be welded are pressed together to make the joint. When metal is melted to fluid state another metal is filled in the region of the joint and allowed to cool to solidify.

Connections between metal plates, angles, pipes, and other structural elements are frequently made by welding. Fusion welds are made by melting portions of the materials

Metals may also be joined by resistance welding in which a small area or spot is heated under high localized pressure. The material is not melted with this type of welding. Other joining methods for metals include brazing and soldering, in which the joining metal is melted but the parts to be connected are not melted. Such connections are usually much weaker than the materials being connected. Fusion welding is the most effective method when high strength is an important factor, and it will be discussed more in detail.

At present time welding has become a powerful technology and almost all joining of steels is done by welding. Welding has replaced riveting, particularly in the manufacturing of boilers and ships, and in many cases is being preferred in construction of structure. Some of the advantages of welding over riveting are as follows :

- The plates and sections to be joined are not weakend as happens in case of riveting. For riveting drilling or punching removes the metal from working sections thus making them weaker. The net weight of metal making the joint is less in case of welded joints. The weight added due to filling of metal is much less than the weight added by way of riveting. The butt welded joints do not require any cover keeping the weight low.

- The riveted joints require a great deal of labour in marking and making holes. There is no possibility of making the riveting process automated whereas welding has become fully automatic particularly when long seams, such as in boiler are to be produced.
- Tight and leak proof joints are ensured by welding.
- Welding is a noiseless operation whereas riveting can never be noiseless
- Curved parts are easily joined by welding

However, following difficulties in producing good welded joints must be kept in mind.
Some of these may become disadvantage of welding unless special care is taken.

- The parts to be joined have to be prepared carefully along the seam and arranged to have sufficient clearance so that filler metal can easily be filled.
- Since metal is heated to a very high temperature (to melting point in most cases) there exists a strong possibility of metallurgical changes taking place in parent metals, particularly in the close vicinity of the joint. These changes may deteriorate the mechanical properties. The loss of ductility is a major problem.
- Since the metal to be joined is held by clamping, residual stresses develop in the region of weld. These residual stresses are often tensile in nature and greatly affect the behavior of metal under fatigue loading.
- The quality of weld is highly dependedent upon the welder if automated process is not used.
- The residual stresses may be removed and metallurgical changes reversed by heat treatment (annealing and normalising). But very large structures are difficult to heat.

- Stress concentration is produced where filler metal joins with the parent metal. Care must be taken in post welding clearing and grinding of joint to eliminate such stress concentration.

- The welded joints are particularly found to lose their ductility at low temperature. Combination of possible existence of defects, stress concentration and loss of ductility has been the reason of various structural failure in ships, reservoirs, pressure vessels and bridge structure.
- The designer has to keep above points in mind while designing welded joins. Recommendation about treatment, grinding welds and inspection for defects must be thoroughly incorporated in design. Further, in non-automated processes the welder should be made to undergo rigorous skill tests before he is put on a job.

Types of Welded Joints, Strength

Two types of welding joints are clearly recognized viz. joints between two plates that overlap and joints between two plates that butt with each other. Figure 4.1(a) shows a fillet joint and Figures 4.1(b) and (c) show example of butt joints. Four types of fillet joints commonly used are illustrated in Figure 4.2.

It may be of interest to note that if a weld is analyzed elastically the shearing stress distribution in the weld turns out to be as shown in Figure 4.3(a). The stress is much higher at ends but quickly reduces to constant minimum. But in actual practice the ends of the weld deform plastically making distribution almost uniform. This is true if weld is ductile which is normally true. Thus, in the design of a welded joint it is reasonable to assume uniform distribution of shearing stress. However, the fillet welds are designed on the assumption that failure will occur by shearing the minimum section of the weld. This minimum section is called the throat of the weld and is shown as section *AB* in Figure 4.3(b). This is true in case of both parallel and transverse welds as shown in Figure 4.2 and is supported by experiments.

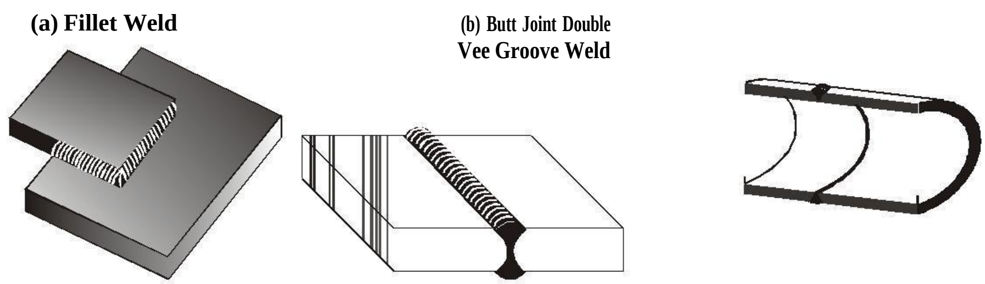
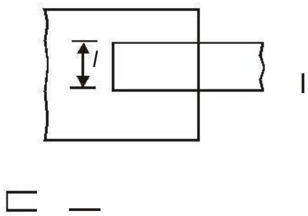
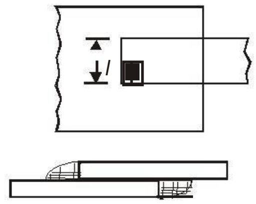
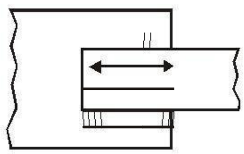


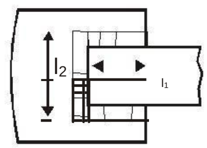
Figure 4.1 : Welded Joints





(a) Single Fillet Joint (Transverse) **(b) Double Fillet Joint (Transverse)**





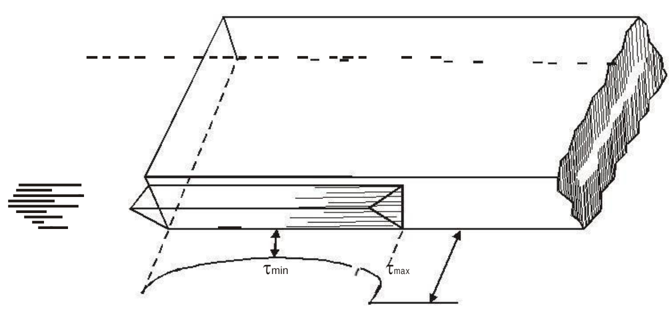
(c) Parallel Fillet Joint **(d) Compound Fillet Joint**

Figure 4.2 : Types of Fillet Welded Joints

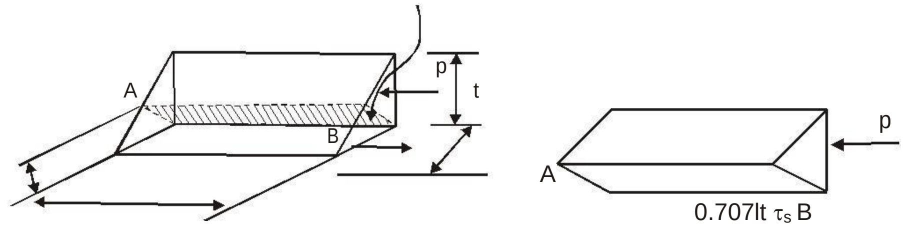
Although it is desirable to make fillet weld slightly concave, yet a reinforced weld obtained from welding is ground to obtain a triangular section with two sides equal to the thickness of plates jointed. Therefore, the width of the section *AB* in Figure 4.3(b) will be 0.707 *t* and area over which shearing will occur, for a length *l*, will be 0.707 *t l* and force of resistance will be 0.707 $t\, l\, \tau$ as shown in Figure 4.3(c). Here τ_s is the permissible shearing stress. The permissible shearing stress is chosen as 50% of permissible tensile stress of parents metal for manual welding. In case of automated process the permissible shearing stress in the weld is assumed as 70% of permissible tensile stress of parent metal. If the load on the joints varies between P_{min} and P_{max}, the permissible stress is multiplied by a factor γ,

ting on a fillet welded joint of length *l*, is

$$P = 0.707\, t\, l\, \tau_S \qquad \ldots (4.2)$$



(a)



(b) **(c)**

Figure 4.3

This equation will be used to calculate the design dimension l

$$l = \frac{P}{0.707} t \tau_s \qquad \ldots (4.3)$$

This length is increased by 12.5 mm to take care of starting of weld in each segment.

T-Joint

It is a case of fillet weld but a plate is welded at right angle to another. The joint may be subject a tension, P or bending due to P acting parallel to weld, as can be seen in Figure 4.4. Two loads are shown in this figure for convenience but they will be analysed separately.

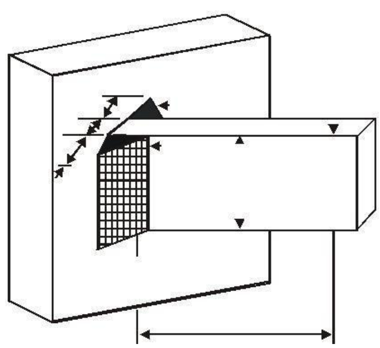


Figure 4.4 : A T-Joint under Axial and Eccentric Load

Axial Tension

It is a case of fillet weld. The leg of the weld is equal to thickness of the plate, t and the cross-section of fillet is an isosceles triangle. Thus the depth of the throat of the weld is 0.7 t as in last section. The length of the weld is l and hence the areas to resist shearing failure

$$A = 0.7\, t\, l \times 2$$

$$\therefore \qquad P = 1.4\, t\, l\, \tau_s \qquad \ldots (4.4)$$

where τ_s is the permissible shearing stress in the weld.

Eccentric Load

Figure 4.4 shows an eccentric load P with an eccentricity of e which is measured as distance between line of action P and the line joining the centers of gravity of triangular sections of fillet welds on two sides. The load P will have two actions on the fillet, viz. :

induce shearing along the throat plane, and

causes bending of throat plane of the weld.

The shearing stress due to P acting as a shearing force will be induced on the area

$$A = 0.7\, t\, l \times 2$$

$$P = 1.4\, t\, l\, \tau \qquad \text{. . . (4.5)}$$

Fore calculating bending stress, one has to consider modulus of the section of the throat plane. This section has a width of 0.7 t and depth equal to l. There are two such sections. Though both these sections are not perpendicular to the axis of the joint this fact is disregarded and modulus of section is calculated as if these sections were perpendicular to the axis.

Calling modulus of section, Z

$$Z = 2 \times \frac{1}{6} 0.7\, t\, l^2$$

Bending moment $\qquad M = P\,.\, e$

Bending stress $\qquad \sigma = \frac{M}{Z} = \frac{6Pe}{1.4 t l^2} \qquad \text{. . . (4.6)}$

The value of bending stress σ occurs at top of the fillet at point A in Figure 4.4. At these points the shearing stress is given by Eq. (4.5). The maximum shearing stress can be found by using the formula,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Unsymmetrical Section Loaded Axially

Figure 4.5 presents an angle section welded to a plate. If a tensile force P is applied so as to pass through the centre of gravity of the section then the length of the fillet nearer to CG (l_b) will take greater proportion of the force P than the length of fillet weld which is away from

CG. The lengths l_a and l_b are to be so proportioned that the forces carried by two fillet welds exert no moment about centre of gravity axis. The two fillets are at distances of *a* and *b* respectively from CG (see Figure 4.5) and if *S* is the force per unit length carried in the welds, then

$$S\, l_a\, a - S\, l_b\, b = 0$$

or $$\frac{l_a}{l_b} = \frac{b}{a}$$

or $$\frac{l_a + l_b}{l_b} = \frac{l}{l_b} = \frac{a + b}{a}$$

$$l_b = \frac{al}{a + b} \quad \text{and} \quad l_a = \frac{bl}{a + b} \qquad \ldots (4.8)$$

Where, $l = l_a + l_b$.

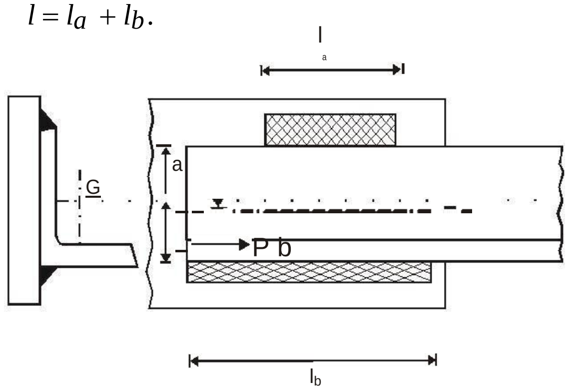
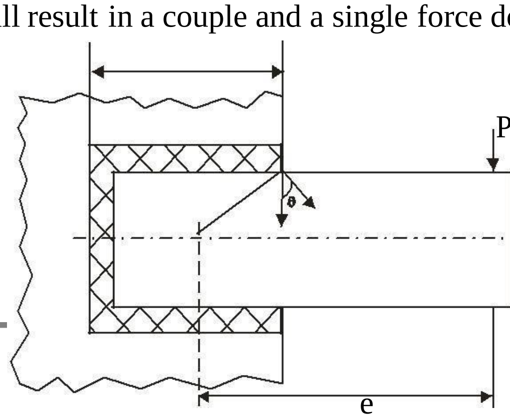


Figure 4.5 : An Unsymmetrical Section Welded to Plate and Welded along Line of CG

Eccentrically Loaded Welded Joint

We have earlier talked of a *T*-joint under axial load. Another example of eccentric loading is shown in Figure 4.6. In this case the force applied at a distance from the CG of the weld group is in the plane of the fillet weld. This will cause the torsional effect in the same way as was considered for riveted joint in Figure 3.16. Two equal and apposite forces are assumed at CG, each being equal to *P* will result in a couple and a single force downward at *G*.



This force P at G will cause what we term as primary shearing stress denoted by τ_1.

Here t is the thickness of the plate. The torque P . e will cause secondary shearing stress τ_2 at the weld end, A, which will be greater than shearing stress at any other point in the weld, A is at a distance of R from CG. It can be shown that

$$\tau_2 = \frac{P \cdot e \cdot R}{J_G} \qquad \ldots (4.10)$$

Here, J_G is the polar moment of inertia of fillet weld about G. AS shown in Figure 4.6, τ_1 and τ_2 at A act at an angle Q. Since nature of both these stresses is same, they can be added vectorially. The resultant stress τ_A is

$$\tau_A = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \cos\theta} \qquad \ldots (4.11)$$

To solve a problem of eccentric loading such as one depicted in Figure 4.6 one would require computing the value of J_G. It is generally required to find the size of the weld, t, whereas the width involved in calculation of J_G is that of throat of the weld, say h. It is also known that h = 0.707 t so that the section of weld is an isosceles triangle. In a given problem the length a is given which is the distance from point A to the inner side of the vertical fillet while distance b will be equal to the width of the plate or distance between inner edges of horizontal fillets. To compute J_G following procedure is adopted.

(a) Determine the position of CG of weld group with reference to inside edges of vertical and horizontal fillets.

(b) Determine second moment of inertia I_{xx1} and I_{yy1}, respectively first with respect to horizontal and vertical axes passing through their CG and then transferring them to horizontal and vertical axes passing through CG of weld group.

(c) Similarly for vertical fillet determine and transfer I_{xx2} and I_{YY2} to the horizontal and vertical axes passing through CG of weld group.

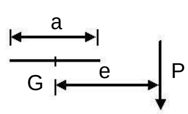
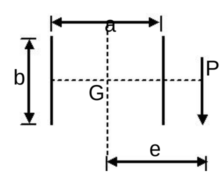
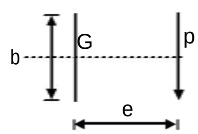
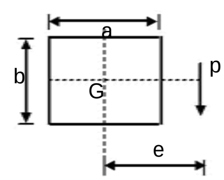
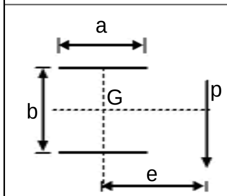
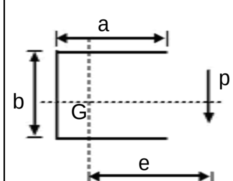
(d) By adding the four moments of inertia obtain J_G, i.e.

$$J_G = I'_{xx1} + I'_{yy1} + I'_{xx2} + I'_{yy2}$$

where quantities on right hand side refer to horizontal and vertical axes through CG of weld group.

One important point if considered during calculation will greatly help achieve solution. The expressions for I_{xx1}' , I_{yy1}' , etc. will contain terms like ah^3 and ha^3, etc. Since h is much smaller than a or b, its cube can be neglected. This will greatly reduce the efforts on computations. Finally h may be replaced by $0.707\ t$. The procedure will be illustrated in one of the following solved problems. Table 4.1 gives formulae for J_G in different weld groups.

Table 4.1 : Polar Moment of Inertia of Weld Groups about Axis Passing through Centre of Gravity

Weld Group	J_G	**Weld Group**	J_G
	$\frac{ha^3}{12}$		$\frac{hb(3a^2+b^2)}{6}$
	$\frac{hb^3}{12}$		$\frac{h(a+b)^3}{6}$
	$\frac{ha(3b^2+a^2)}{6}$		$h\,\frac{8a^3+6ab^2+b^3}{12} - \frac{ha^4}{2a+b}$

Problem:

A steel plate strip of 150 mm width and 10 mm thickness is welded by a compound fillet weld to another plate. The strip is required to carry an axial load P such that P is equal to tensile load capacity of the strip with a factor of safety of 2.5 on ultimate tensile strength of strip. Calculate the length of the fillet weld and show on diagram. Ultimate tensile strength of strip material is 380 MPa. Find fillet length if $P_{min} = P_2$ and $P_{max} = P$.

Solution

This is the type of the joint shown in Figure 4.2(d). The permissible tensile stress in strip material is $380/2.5 = 152$ MPa . Hence permissible shearing stress in the weld, with the assumption of manual welding,

$$\tau_s = \frac{1}{2} \times 152 = 76 \text{ MPa}$$

The axial load on the joint as shown in Figure 4.7

$$P = (150 \times 10)\, \sigma_t = 1500 \times 152 = 228000 \text{ N}$$

Also $$P = 228000 = 0.707 \times 10\, (2l_1 + l_2) \times 76$$

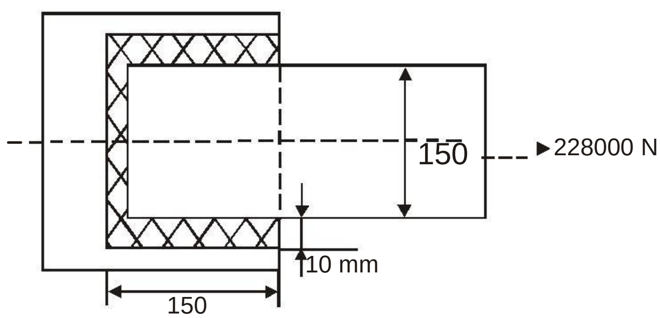


Figure 4.7

Apparently the vertical weld length, $l_2 = 150$ mm

$$\therefore \quad l_1 = \frac{228000}{2 \times 0.07 \times 76} - \frac{150}{2} = 137.16 \text{ mm}$$

Correct l_1 by adding 12.5 mm, $l_1 = 137.16 + 12.5 = 149.66$ say 150 mm.

When P varies between $P_{min} = \frac{P}{2}$ and $P_{max} = P$, the factor

$$\gamma = \frac{1}{4 - 1 P_{min}} = \frac{1}{4 - 1 \cdot 1} = \frac{1}{8 - 1} = \frac{6}{7}$$

$$3P_{max} \quad 3$$

$$\therefore \qquad \tau_s = \frac{6}{7} \times 76 = 65.143\ \text{N/mm}^2$$

$$\therefore \qquad l_1 = \frac{228000}{2 \times 0.07 \times 65.143} - 75 + 12.5 = 185\ \text{mm}$$

Problem:

A bracket is welded to its support as shown in Figure 4.8. All welds are fillet welds of equal thickness. Determine the fillet size if the permissible stress in the weld is 80 N/mm^2

The welded joint of Figure 4.8 is eccentrically loaded. The weld will be subjected to two types of shearing stress. The direct primary shearing stress τ_1 due to shearing force acting through CG. The shearing force is 25 kN, through CG of weld group. The secondary shearing stress τ_2 due to torque $P.e$ acting about CG.

Position of CG

The weld group is made up of three fillet weld, 1, 2, and 3 which are regarded as rectangles of width *h* and lengths respectively 60 mm, 60 mm and 150 mm. With *o* (middle of vertical fillet of height 150 mm) *ox* and *oy* are drawn horizontal and vertical axes. Areas of fillets 1, 2 and 3 are denoted by A_1, A_2 and A_3 so that

$A_1 = 60\ t$, $A_2 = 60\ t$, $A_3 = 150\ t$ mm^2

Since the weld group is symmetrical about horizontal axis, CG will be on *ox* axis. Its distance from *oy* is $\bar{x}$. Then

$$\bar{x} = \frac{A_1 (30) + A_2 (30) + A_3 (0)}{A_1 + A_2 + A_3}$$

i.e. $$\bar{x} = \frac{60\,h \times 30 + 60\,h \times 30}{60\,h + 60\,h + 150\,h} = \frac{2 \times 60 \times 30}{270}$$

$$= 13.33\ \text{mm}$$

Remember $h = 0.707\ t$.

Polar Moment of Inertia of Weld Group about CG

I_{xx} and I_{yy} denote second moments of inertia of plane rectangles about horizontal and vertical axes passing through CG of respective rectangles. $I_{xx'}$ and I'_{yy} denote moments of inertia about horizontal and vertical axes passing through CG of weld group, *G*. The suffixes 1, 2 and 3 will be used to distinguish between areas as 1, 2 and 3. It may also be noted that since h is much smaller than and also not known at the initial stage, it may be neglected in comparison with transfer distances 13.33 mm and 75 mm, etc.

$$I_{xx2} = \frac{60\,h^3}{12} = 5h^3$$

$$I_{xx3} = \frac{h\,(150)^3}{12} = 281.25 \times 10^3 h$$

$$I'_{xx1} = 5h^3 + 60h\,(75)^2 = 5h^3 + 337.5 \times 10^3\,h$$

$$I'_{xx2} = 5h^3 + 60h\,(75)^2 = 5h^3 + 337.5 \times 10^3\,h$$

$$I'_{xx3} = I_{xx3} = 281.25 \times 10^3 h$$

$$I_{yy1} = \frac{h\,(60)^3}{12} = 18 \times 10^3\,h$$

$$I_{yy2} = \frac{h\,(60)^3}{12} = 18 \times 10^3 h$$

$$I_{yy3} = \frac{150 \times (h)^3}{12} = 12.5\,h^3$$

$$I'_{yy1} = 18 \times 10^3\,h + 60h\,(16.7)^2 = 34.7 \times 10^3\,h$$

$$I'_{yy2} = 18 \times 10^3\,h + 60h\,(16.7)^2 = 34.7 \times 10^3\,h$$

$$= 12.5h^3 + 120\,h \times (13.33)^2 = 12.5\,h^3 + 21.32 \times 10^3\,h$$

The polar moment of inertia of the weld group about CG.

$$J_G = I'_{xx1} + I'_{xx2} + I'_{xx3} + I'_{yy1} + I'_{yy2} + I'_{yy3}$$

$$= 5\,h^3 + 337.5 \times 10^3\,h + 5h^3 + 337.5 \times 10^3\,h + 281.25 \times 10^3\,h$$

$$\pi\ 34.7 \times 10^3\,h + 34.7 \times 10^3\,h + 12.5\,h^3 + 21.32 \times 10^3\,h$$

$$\tau\ (22.5h^3 + 1047 \times 10^3\,h)\ \text{mm}^4$$

Since h is a small quantity, contribution of its cubes, h^3, may be neglected. Hence to the first approximation,

$J_G = 1047 \times 10^3\,h\ \text{mm}^4$ . . . (iii)

The secondary shearing stress at any point on the fillet at a distance of R from G is given as

$$\tau' = \frac{P \cdot eR}{J_G}$$

This stress will act perpendicular to *R*. Obviously its value will be higher if *R* is higher. *R* at *A* and *C* is greatest. From triangle *ABG*.

$$R_A = \sqrt{(AB^2 + BG^2)}$$

$$= \sqrt{(75^2 + 46.7^2)} = 88.5 \text{ mm}$$

$$e = 50 + BG = 50 + 46.7 = 96.7 \text{ mm}$$

for τ' and calling its value at *A* as τ_2

$$\tau_2 = \frac{25000 \times 96.7 \times 88.35}{1047 \times 10^3\, h} = \frac{204}{h} \text{ MPa} = \frac{288.5}{t} \text{ MPa} \quad . . . \text{(iv)}$$

$$\cos\theta = \frac{BG}{R_A} = \frac{46.7}{88.35} = 0.53$$

Since τ_1 and τ_2 are of same type they can be added vectorially to obtain resultant
stress.

$$\tau_A = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2\cos\theta}$$

$$= \frac{1}{t}\sqrt{(131)^2 + (288.5)^2 + 2 \times 131 \times 288.5 \times 0.53}$$

$$= \frac{1}{t}\sqrt{17161 + 832.5 + 40067} = \frac{374.8}{t} \text{ MPa}$$

The stress τ_A should not exceed 80 MPa, the permissible value

$$\therefore \qquad \frac{374.8}{t} = 80$$

$$\therefore \qquad t = \frac{374.8}{80} = 4.685 \text{ say } 5.0 \text{ mm}$$

UNIT 8

POWER SCREWS

Instructional Objectives

- *Power screw mechanism.*
- *The thread forms used in power screws.*
- *Torque required to raise and lower a load in a power screw*
- *Efficiency of a power screw and condition for self locking*
- *Describe geometry of screw and nut,*
- *Determine forces on screw and nut threads,*
- *Calculate dimensions of screw and nut for transmission of force, and*
- *Find the force on screw fastener and load transmitted to parts jointed by fasteners.*

Screws are used for power transmission or transmission of force. A screw is a cylinder on whose surface helical projection is created in form of thread. The thread will have specified width and depth, which bear some ratio with the diameter of the cylinder. The screw rotates in a *nut*, which has corresponding helical groove on the internal surface. Thus a nut and a screw make a connected pair in which one remains stationary while other rotates and translates axially. The helical surface of the screw thread makes surface contact with the helical groove surface of the nut. If an axial force acts on, say screw moving inside stationary nut, the point of application of the force will move as the screw advances in axial direction. This will result in work being done and hence power being transmitted. Both types – one in which screw rotates and advances in a stationary nut or one in which screw rotates between fixed support and nut is free to move axially – are used in practice. In the latter case the force acting on nut will move as nut translates. However, the friction between the surfaces of contact will require some power to be overcome. Hence the power delivered by the screw-nut pair will be less than the power supplied.

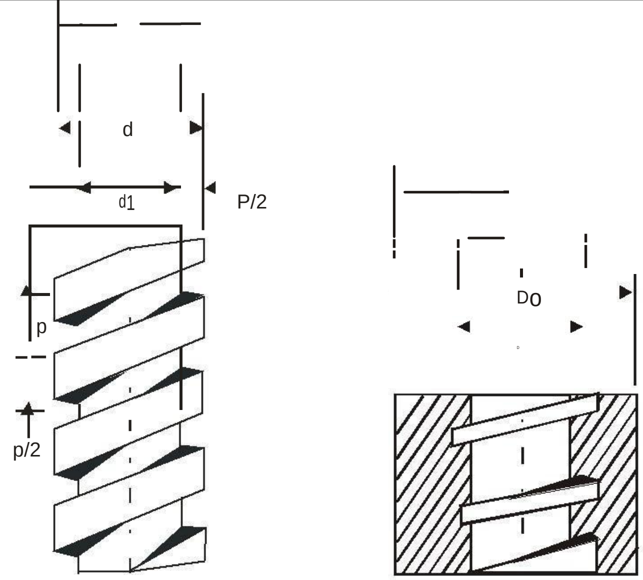
The contact surfaces of screw thread and nut groove are made perpendicular to the outside and inside cylindrical surfaces. They are sometimes given a small inclination. Such provision keeps coefficient of friction to a reasonable low level. The coefficient of friction may be further reduced by lubrication. However, by creating considerably inclined surfaces in nut and screw the effective coefficient of friction is increased. Such screw thread joint will make advancing of threaded part difficult. This combination will be used as fastening device.

Geometry of Thread

Look at Figures 8.1(a) and (b) and you will get a fair idea how would a screw and a nut appear. The screw will pass into nut by rotating either of them. For understanding how a helical thread can be formed on a cylinder you can take a plane sheet of paper and draw an inclined line on it. Then roll the paper to form a cylinder by bringing two opposite edges of the paper together.

The line which you drew will appear like a helix on the surface of the cylinder. A line drawn as *AC*, inclined at angle α with horizontal line AA_1, will be wrapped on the cylinder to AA', looking like a helix. AA_1, becomes the circumference of the cylinder and A' coincides with *C*. In Figure 5.2, you can see two parallel lines *AC* and $A'C$ drawn inclined at α to horizontal and then paper wrapped to form a cylinder and thus two threads are formed on cylinder. *p* is the vertical distance between *A* and *C* or between *C* and *C*. If the paper is wrapped such that the lines drawn are on the inside surface, you can get the idea of internal thread.

The distance A1C which is equal to AA' and A″C″ is called the pitch of the screw. Pitch is apparently the distance between two corresponding points on two consecutive threads. The angle α between the base of the triangle and hypotenuse becomes the angle of helix. Obviously,



(a) A Screw **(b) A Nut**

Figure 8.1 : Screw and Nut with Helical Surfaces Cut on Outside and Inside Surfaces of Cylinders, respectively

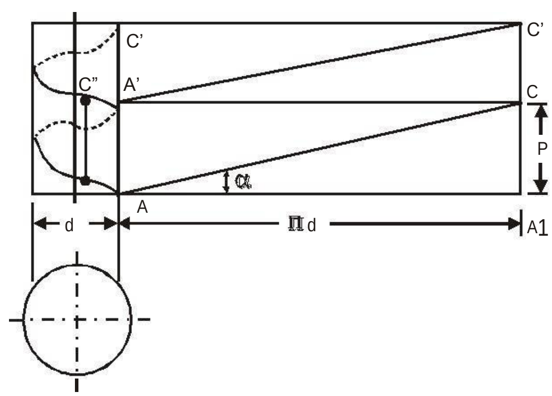


Figure 8.2 : Formation of a Helix on the Cylindrical Surface

$$P = \pi \text{ d} \tan \alpha$$

$$\tan \alpha = \frac{P}{\pi d}$$

The thread depicted in Figure 8.1(a) is a square thread. The Vee thread is created by a triangular section while trapezoidal thread has a trapezium section. This thread is also known as the Acme thread. Buttress thread has a triangular section but one side of the triangle is perpendicular to the axis. The square, the Acme and the buttress threads are used for power transmission, as they are more efficient than the Vee thread. The square thread is most efficient but difficult to produce and hence becomes costly. The adjustment for wear in square thread is very difficult but can be easily achieved in the Acme threads, by splitting the nut along the axis. The Acme threads thus can be used as power transmission element when power is to be transmitted in both the directions.

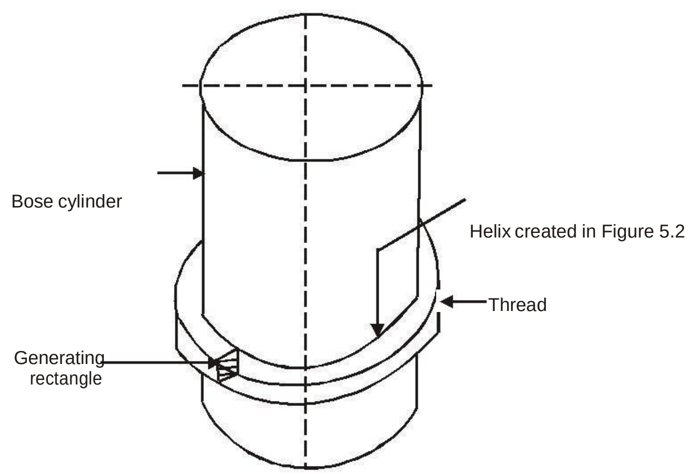


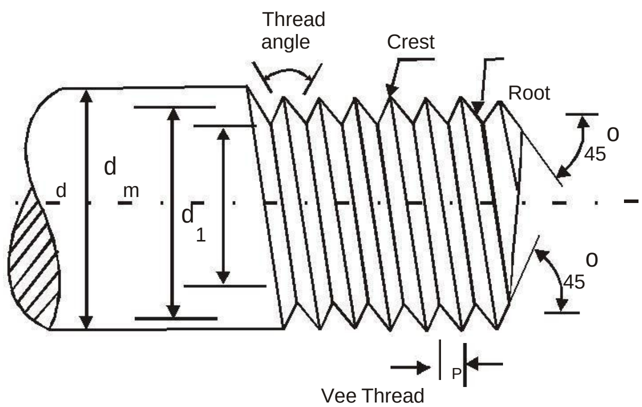
Figure 8.3 : Generating a Square Section Thread on Cylindrical Surface

There is little or no backlash in the Acme threads which are commonly used as feed and lead screws of machine tools. The buttress thread having one side flat and other sloping combines the advantage of square thread and Acme thread. The flat side provides the efficiency of power transmission while the inclined side provides the ease of adjustment.

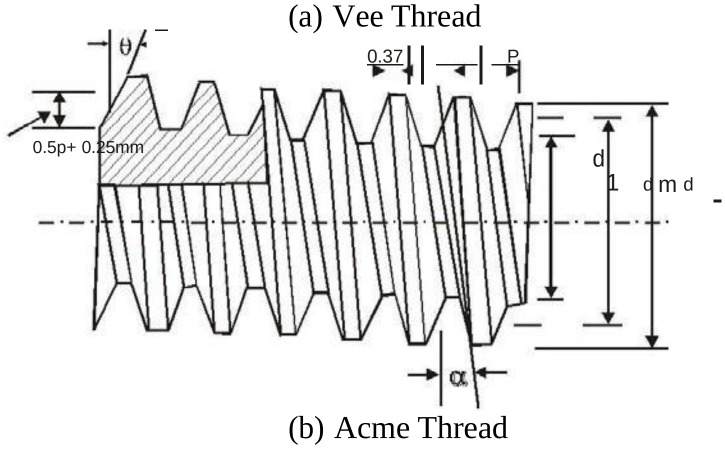
However, these advantages become possible only when the power is transmitted in one direction. Vee threads for their lower efficiency for power transmission are used as fasteners. Due to sides being inclined the effective coefficient of friction between the screw and the nut increases. Figure 8.4 shows Vee or triangular, the Acme and the buttress threads with leading nomenclature.

The major diameter is the largest diameter of the screw thread denoted by *d* for external thread and by *D* for internal thread. *Minor diameter* (d_1 or D_1) is the smallest diameter of the screw. Some times more than one thread may be cut on the screw. These multiple threads may be easily seen at the end of the screw where more than one thread will appear to start. Multiple start threads give the advantage that screw can move through a longer distance in the nut when given one rotation as compared to the screw with a single thread or start. The distance moved by a screw along its axis when given one rotation is called the *lead*. Apparently

$$\text{lead} = \text{number of starts} \times \text{pitch}$$



(a) Vee Thread



(b) Acme Thread

P/2 45 o P 0.245P

0.503P 0.89P

d dm d 1 0.139P

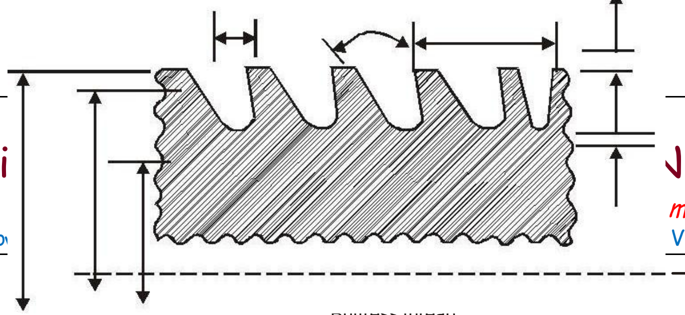
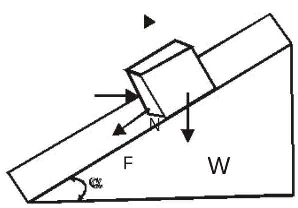


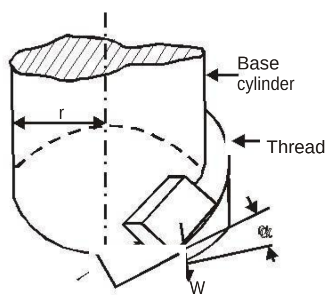
Figure 8.4 (c)Buttress Thread

Mechanics of Screw and Nut Pair

It was mentioned earlier that the motion between nut and screw is like a body moving on an inclined plane. Figures 8.5(a) and (b) will explain this motion. The body of weight *W* is pushed up the inclined plane by a force *P* which acts upon the body horizontally. This inclined plane is bent round a cylinder in Figure 8.5(b) and aome body is being pushed up the plane while force *P* remains horizontal but also tangential to circular path of the body. This illustrates how the motion of the nut on the thread is similar to motion of a body on an inclined plane. The weight of the body on the inclined plane is replaced by weight carried by the nut in axial direction. The force *P* is applied by the help of a wrench and *W* may be the reaction developed between surfaces of contact.



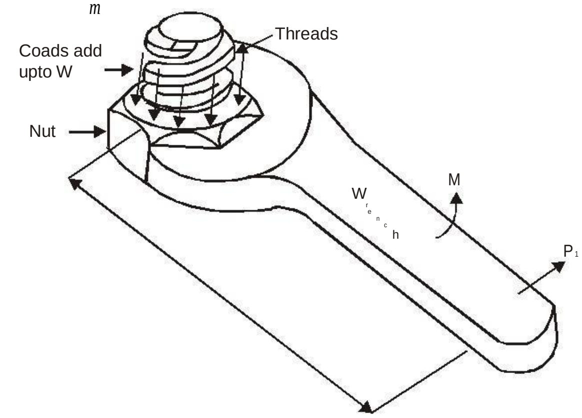
(a) Inclined Plane



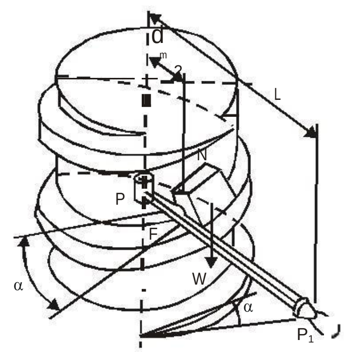
(b) Inclined Plane Wrapped Round a Base Cylinder to Form a Thread Figure 8.5

This is illustrated in Figures 8.6(a) and (b). Imagine that a lever is pivoted on the axis of the cylinder and pushes the body of weight *W* up the incline of helically wrapped plane on the cylinder. The lever touches the body at a radius *r* while a force P_1 is applied on the lever at an arm length of *L*. In the nomenclature we have already defined the outer diameter of the thread as major diameter, *d* and the smallest diameter as the diameter of the cylinder, *d*1, (also called core diameter). *r* is *dm* where *dm* is the mean diameter of 2 thread, being mean of *d* and d_1. If *P* be the force applied by the lever on the body of weight W, then by taking moments of forces, acting upon the lever, about the axis of the cylinder.

$$P = \frac{2P_1 L}{d_m} \qquad \ldots (8.3)$$



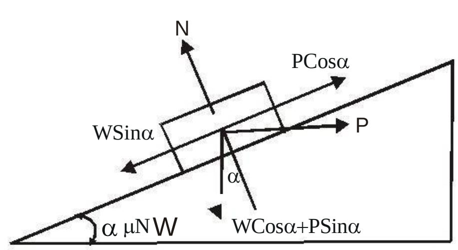
(a)



(b)

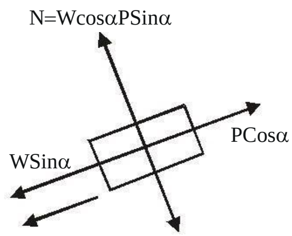
Figure 8.6

The force *P* has to overcome the friction as well as cause lifting of the body in vertical direction. To find the relationship between force *P*, called effort, and weight of the body, *W*, we have to consider the equilibrium of the body on an inclined plane as shown in Figures 5.7(a) and (b). The free body diagram clearly shows forces along and perpendicular to the inclined plane.



(a) A Body being Pushed Up the Inclined Plane with Angle of Inclination *a*, by a Horizontal

Force P



(b) Free Body Diagram of Body of Weight *w*

Figure 8.7

The sum of the forces perpendicular to the plane and sum of the forces along the plane should separately be zero to satisfy the conditions of equilibrium. μ is taken as coefficient of friction between the body and the plane, which is same as coefficient of friction between the nut and screw thread surfaces. $\mu = \tan\phi$ where ϕ is the angle of friction. Summing up the forces perpendicular to the plane.

The normal reaction,

$$N = W\cos\alpha + P\sin\alpha$$

Hence, force of friction between the surfaces of contact

$$F = \mu N = \mu W\cos\alpha + \mu P\sin\alpha$$

Summing up the forces parallel to the inclined plane

$$P\cos\alpha = W\sin\alpha + \mu W\cos\alpha + \mu P\sin\alpha \qquad \ldots \text{(i)}$$

Replacing μ by $\tan\phi = \dfrac{\sin\phi}{\cos\phi}$

$$P\cos\alpha - \frac{\sin\phi}{\cos\phi} P\sin\alpha = W\sin\alpha + \frac{\sin\phi}{\cos\phi} W\cos\alpha$$

$$p\cos\alpha\cos\phi - P\sin\alpha\sin\phi = W\sin\alpha\cos\phi + W\cos\alpha\sin\phi \qquad \ldots \text{(ii)}$$

$$P\cos(\alpha+\phi) = W\sin(\alpha+\phi)$$

$$P = W\tan(\alpha+\phi) \qquad \ldots (8.4)$$

If there is no friction, $\phi = 0$ and effort in such a case is called *ideal effort* denoted by P_i where

$$P_i = W\tan\alpha \qquad \ldots (8.5)$$

Hence the efficiency of an inclined plane with inclination of screw having helix angle□ is

$$\eta = \frac{P_i}{p}$$

$$= \frac{\tan \alpha}{\tan (\alpha + \phi)}$$

If in a situation as shown in Figure 8.7(a), P is removed, will the body slide down obviously it will depend upon the fact as to how large angle α is. If $W \sin \alpha > \mu W \cos \alpha$ the body will slide down under its own weight (Examine (i) and (ii) with $P = 0$). Same thing will happen in case of a nut in Figure 8.6 (a), i.e. when effort P_1 is removed from the wrench or wrench is removed, the nut will rotate back under load W. It means the nut is not self-locking. However, if α is reduced it can be seen that at $\alpha = \phi$, the downward component (along the plane) of weight W, i.e. $W \sin \alpha$ and friction force (along the plane) $\mu W \cos \alpha$ become equal and the body remains just stationary or the nut does not move down. If $\alpha < \phi$, the body will need a force to act so as to push it down. If this force is P' then

$$P' = W \tan (\phi - \alpha) \qquad \ldots (8.7)$$

Naturally screws of $\alpha > \phi$, will not be self locking or in other words they cannot act as fasteners. If, however, the angle $\alpha < \phi$, an effort P', given by Eq. (8.7) will be required to unscrew the nut, and such screws can be used as fasteners.

The Eq. (8.6) which defines the efficiency of the screw and that the condition for screw to be self locking is that $\alpha \leq \phi$ can be used to determine the maximum efficiency of a self locking screw and nut pair.

For self locking condition, the efficiency

$$\eta_s \leq \frac{\tan \phi}{\tan (\phi + \phi)}$$

$$\leq \frac{\tan \phi}{\tan 2\phi} \leq \frac{\tan \phi \,(1 - \tan^2 \phi)}{2 \tan \phi}$$

$$\leq \frac{1 - \tan^2 \phi}{2}$$

Since $\tan^2 \phi$ is always less than 1, ($\mu = \tan \phi$)

$$\eta_s < \frac{1}{2} \qquad \ldots (8.8)$$

The screw having efficiency greater than 50% is said to over haul, meaning the load W will cause the nut to roll down.

POWER SCREW MECHANICS

In the preceding section the simple case of a square thread was considered. As will be seen in the text that follows that the square thread is more efficient than the Acme thread because in the Acme thread the effective coefficient of friction increases, yet for power screws it is the Acme thread which is used more predominantly. The Acme thread can be machined more easily than the square thread and more importantly the clearance in the Acme thread can be adjusted to take care of the wear or machining inaccuracy.

Figure 8.8 shows the nuts in pair with square and the Acme threads and an adjusting mechanism for the acme thread.

An Acme thread has two inclinations. Firstly the plane of the thread is sloping along angle of helix in the direction of the helix. The plane of the thread also slopes away from the circumference of the screw, i.e. the circumference of diameter d_1, The same is true for the V-thread. Both types of threads are as shown in Figures 8.4(a) and (b). The effect of inclination in the radial direction is to increase the normal reaction between the nut and the screw. This inclination in the radial direction of thread gives a shape of trapezium of angle 2θ as shown in Figure 8.9 and since the motion will occur perpendicular to the plane of paper; the force of friction will depend upon the normal reaction.

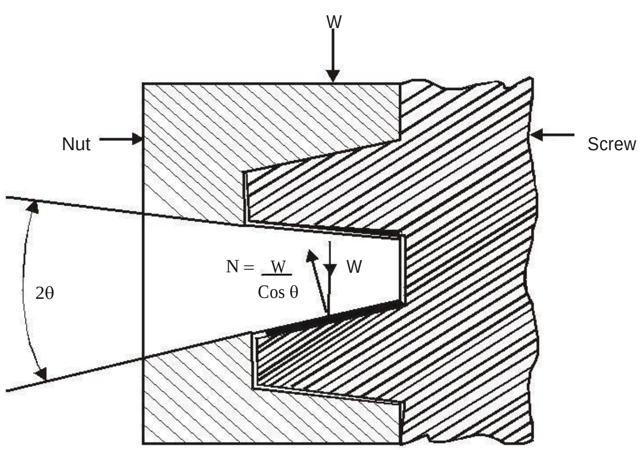


Figure 8.9 : A Nut Moving on a Screw having Acme Thread

For a vertical force W pressing the nut on the thread of screw, the normal reaction is N.

Resolving N in the vertical direction and equating with W

$$N \cos \theta = W \text{ or } N = \frac{W}{\cos \theta}$$

and hence the force of friction along the direction of helix is μN or $\frac{\mu W}{\cos \theta}$ which can also be written as $\mu' W$ and μ' can be called a modified or effective coefficient of friction. No doubt you can see that

$$\mu' = \frac{\mu}{\cos\theta} > \mu \qquad \ldots (8.9)$$

It is because $\cos\theta$ is less than 1. Greater the angle θ, lesser the $\cos\theta$ and hence μ' will increase with increasing θ. This is what happens in V-thread. The force of friction between nut and thread in V-threads is greater than in Acme thread. The *P*-*W* relationship given by Eq. (8.4) stands valid for square thread and can be modified for Acme thread by replacing ϕ by ϕ' where

$$\phi' = \tan^{-1} \mu'$$

$$P = W \tan (\alpha + \phi) \qquad \ldots (8.10)$$

Efficiency of the Acme thread will be

$$\eta' = \frac{\tan \alpha}{\tan (\alpha + \phi)} \qquad \ldots (8.11)$$

A word about the horizontal component of *N*, which is $N \sin \theta$ will be in order. Remember we are talking about the thread round the circumference of the screw. There is other side of the screw on right of Figure 8.9 and $N \sin \theta$ there will be acting to the right. Thus the horizontal components of *N* are balanced.

Here ϕ' is the effective angle of friction which could be $\phi = \tan^{-1} \mu$ if the square thread is on the screw. Apparently the torque in Eq. (8.12) will twist the cylinder of screw and cause shearing stress in it. The cylinder is acted upon by an axial compression also. The axial compressive force causes compressive stress at any point in the section.

Problem:

A square threaded screw is required to work against an axial force of 6.0 kN and has following dimensions. Major diameter d = 32 mm; pitch p = 4 mm with single start, μ = 0.08. Axial force rotates with the screw. Calculate:

(a) Torque required when screw moves against the load.
(b) Torque required when screw moves in the same direction as the load.
(c) Efficiency of the screw.

Solution:

Remember the relationship between p, d and d_1 which has been shown in Figure 8.1.

$$p = d - d_1$$

But $$d_m = \frac{d + d_1}{2} \quad \text{or } d_1 = 2d_m - d$$

$$p = 2\,(d - d_m)$$

or $$d_m = d - \frac{p}{2}$$

Using d = 32 mm and p = 4 mm

$$d_m = 32 - 2 = 30 \text{ mm} \qquad \text{. . . (i)}$$

The angle of helix is related to the circumference of mean circle and the pitch from description of Section 8.2.

$$\tan\alpha = \frac{p}{\pi d_m} = \frac{4}{\pi 30} = 0.042 \qquad \text{. . . (ii)}$$

$$\therefore \qquad \alpha = 2.4^o \qquad \text{. . . (iii)}$$

and $$\tan\phi = \mu = 0.08 \qquad \text{. . . (iv)}$$

$$\tan\phi = 4.57^o$$

From Eq. (8.12) torque required to move screw against load

$$M_t = W\tan(\alpha + \phi)\frac{d_m}{2}$$

$$= 6 \times \tan(2.4 + 4.57) \times \frac{30}{2} = 6 \times 0.12225 \times 15 \text{ Nm}$$

$$= 11 \text{ Nm}$$

When screw moves in the same direction, is a case in which the body moves down the inclined plane. In this case the fore P' to push down is given by Eq. 5.7. Hence, the torque

$$M'_t = P'\frac{d_m}{2} = W\tan(\phi - \alpha)\frac{d_m}{2}$$

$$= 6 \times \tan(4.57 - 2.4) \times \frac{30}{2} = 6 \times 0.038 \times 15 \text{ Nm}$$

$$= 3.42 \text{ Nm} \ldots\ldots\ldots\ldots\ldots\ldots\text{(vi)}$$

From Eq. (8.6), efficiency

$$\eta = \frac{\tan \alpha}{\tan (\alpha + \phi)} = \frac{0.042}{0.12225} = 0.344$$

or $\eta = 34.4\%$ . . . (vii)

Problem:

If in the Example 8.1, the screw has the Acme thread with thread angle $2\theta = 29^o$ instead of square thread, calculate the same quantities.

Solution:

There is no difference in calculation for square and the Acme thread except that in case of the Acme thread the coefficient of friction is modified and effective coefficient of friction is given by Eq. (8.9).

$$\mu' = \frac{\mu}{\cos\theta} = \frac{0.08}{\cos \frac{29}{2}} = \frac{0.08}{0.968} = 0.0826$$

$$\therefore \qquad \phi' = 4.724^o$$

From Figure 8.2(b) for the Acme thread note that

$$d_m = d - \frac{p}{2} - 0.125 \text{ mm}$$

$$= 32 - 2 - 0.125$$

$$\text{or} \qquad d_m = 29.875 \text{ mm} \qquad \ldots \text{(i)}$$

$$\therefore \qquad \tan\alpha = \frac{p}{\pi d_m} = \frac{4}{\pi \times 29.875} = 0.0426$$

$$\alpha = 2.44^o$$

$$M_t = W \tan(\alpha + \phi') \frac{d_m}{2}$$

$$= 3.0 \times 29.875 \times \tan(2.44 + 4.724) = 89.625 \times 0.126 \text{ kNmm}$$

$$\text{or} \qquad M_t = 11.265 \text{ Nm} \qquad \ldots \text{(ii)}$$

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)} = \frac{\tan 2.44}{\tan(2.44 + 4.724)} = \frac{0.0426}{0.126}$$

$$\text{or} \qquad \eta = 33.8\% \qquad \ldots \text{(iii)}$$

When the screw moves in the same direction as the load, the torque

$$M'_t = \frac{W d_m}{2} \tan(\phi' - \alpha)$$

$$= \frac{6 \times 29.875}{2} \tan(4.724 - 2.44) \text{ kNmm}$$

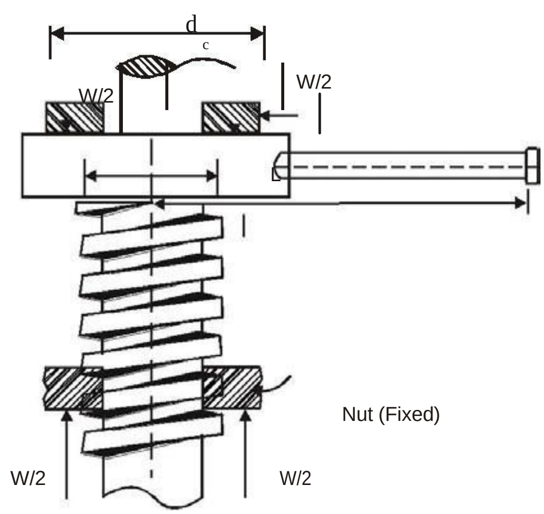
or $$M_t' = 3.58 \text{ Nm} \qquad \ldots \text{(iv)}$$

Comparing the results of Examples 8.1 and 8.2 we can see that the screws have got same major diameter and pitch and for this reason their helix angles are different. Coefficients of friction are inherently different. But the torque on the screw increases by 2.41% and efficiency decreases by 1.744%.

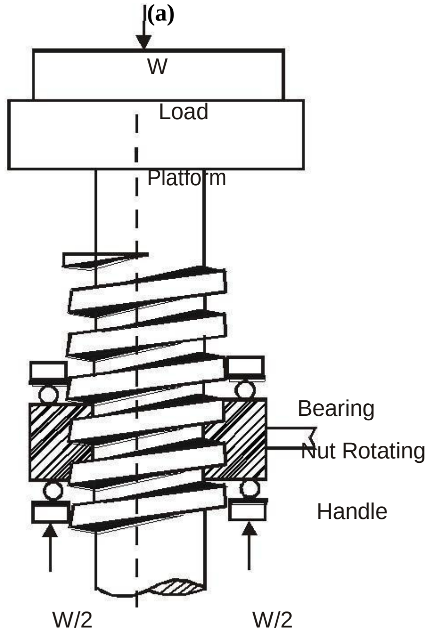
Application of Power Screw

The lead screw of a lathe machine, which moves the tool carriage, is another example of power screw in which the screw rotates in a nut and screw is supported like a shaft between two bearings. The thrust is caused on the nut, which is integral part of the tool carriage. The nut moves along the length of the screw taking the carriage. The reaction of the thrust bears on the supports of the screw. The screw can be used for accurate positioning of the carriage if it is rotated by a separate stepper motor. The screw in transferring of force can also be used in hand operated punching machines, as a lifter of dam gate or as a presser of masses.

If there is a support like a collar, shown in Figure 8.10(a) on the top of which the load is placed so that it does not rotate, then the applied torque has to be equal to the sum of the torque required to rotate the screw in the nut and the friction torque between the surfaces of the collar and load platform. The friction torque between the supporting bearing surface and stationary surface may be reduced by lubrication or by providing rolling bearing as showing in Figure 8.10(b). In any given situation the torque at bearing surface will have to be calculated.



(a)



(b)

Figure 5.10

If the collar surface is like a flat disc of outer and inner diameters of d_o and d_i then the friction torque is given by

$$M_{tf} = \frac{2\mu_c W}{3} \frac{\left(\frac{d_o}{2}\right)^3 - \left(\frac{d_i}{2}\right)^3}{\left(\frac{d_o}{2}\right)^2 - \left(\frac{d_i}{2}\right)^2} = \mu_c W \frac{d_o^3 - d_i^3}{3\left(d_o^2 - d_i^2\right)} \qquad \ldots (8.13)$$

The Eq. (8.13) has not been derived here. In the above equation μ_c is the coefficient of friction between the collar and the platform or between bearing surfaces. Hence M_{tf} can also be writer as $\mu_c W_{rf}$ where r_f is the radius of an imaginary circle. Along the tangent of this circle the force of friction $\mu_e W$ is assumed to act. An approximate value of

$r_f = \frac{d_o + d_i}{4}$ can also be used. Incidentally this value of r_f may be true for unlubricated Surface which is not a reality.

Standard Threads

Standards of threads describe pitch core diameter and major diameter. The standard threads can be cut in standard machine tools with standard cutters and designer can use them for calculation of sizes and ensure interchangeability. We will see in illustrated examples how the standards are used by designer. Presently we describe Indian standard IS 4694-1968 for square threads in which a thread is identified by its nominal diameter which is also the major diameter. According to standard the major diameter of nut is 0.5 mm greater then major diameter of the screw which will provide a clearance of 0.25 mm between the outer surface of screw and inner surface of nut thread. The basic dimensions of square threads are described in Table 8.1.

Table 8.1 : Basic Dimensions of Square Thread, (mm)

Pitch, *p*			5					
Core Dia. d_1	17		19		24		23	
Major Dia. *d*	22		24		26		28	
Pitch, *p*			6					
Core dia. d_1	24		26		28		30	
Major dia. *d*	30		32		34		36	
Pitch *p*			7					
Core dia. d_1	31		33		35		37	
Major dia. *d*	38		40		43		44	
Pitch, *p*			8					
Core dia. d_1	38		40		42		44	
Major dia. *d*	46		48		50		56	
Pitch, *p*			9					
Core dia. d_1,	46		49		51		53	
Major dia. *d*	55		58		60		62	
Pitch, *p*			10					
Core dia. d_1	55	58	60	62	65	68	70	72
Major dia. *d*	65	68	70	72	75	78	80	82

Design of Screw and Nut

Designing is calculating the dimensions, which can be seen in Figure 8.1. They are core diameter, d_i, major diameter *d* and pitch, *p*. the number of threads also has to be determined we have to realize that the load comes upon the screw as axial.

Compression causing compressive stress, which is uniformly distributed over circular cross section of diameter, d_1. As the screw rotates, in the nut it is subjected to a torque given by $P\frac{d_m}{2}$ or $W\tan(\alpha+\phi)\frac{d_m}{2}$. This torque will cause shearing stress, which will be maximum on the surface or at radius of $\frac{d_1}{2}$. The transfer of axial load between the screw thread and nut occurs through surface of thread. The pressure is to be kept within permissible limits, which normally is such that squeezing of oil film between contact surface should not occur. Further the thread and the cylindrical surface of the cylinder may tend to shear off under the load acting on the thread. Lasting we must realize that the axial load on screw makes the screw to act like a column. This column is not allowed to buckle. We will consider each of the above modes of failure to establish equations for calculating dimension.

Direct Stress

The axial load (force) is W, compressive in nature and the area which carries the force is the core cross section of diameter d_i. Hence, compressive stress,

$\sigma = \frac{4W}{\pi d_1^2}$. We will see that this direct compressive stress combines with shearing stress to give principal and maximum shearing stresses. The resulting equations cannot be solved for d_1, hence the expression for σ is used to calculate d_1 from given permissible compressive stress. To account for other stresses, which we will see in next section, the magnitude of the compressive stress is increased by 30%.

Hence $$\sigma = \frac{1.3W}{\frac{\pi d^2}{4}} \qquad \ldots (8.14)$$

As an example if W = 50 kN and permissible compressive stress is 80 MPa, then

$$d_1 = \frac{4 \times 1.3 \times 50 \times 10^3}{\pi \times 80} = 1.0345 \times 10^3$$

By the helps of Table 8.2 you can see that nearest standard value value of d_1 is

33 mm with p = 7 mm and d = 40 mm. This apparently gives all the information we require for a screw but we have to check for safety against other stresses.

Ma imum Shearing Stress

We have already seen that torque $M_t\ W \tan(\alpha + \phi)\ \frac{d_m}{2}$ is required to rotate the screw to cause it to move against force W or to lift weight W. the torque will cause shearing stress in addition to direct compressive stress σ as stated earlier. The shearing stress at any point on surface of core

$$\tau = \frac{16 M_t}{\pi d_1^3}$$

The state of stress at any point on the surface of core of the screw will be compressive or a direct stress σ and a shearing stress τ as shown in Figure 8.11.

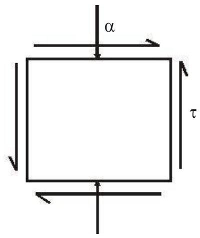


Figure 8.11: State of Stress at any Point on Core Surface of Dia d_1

The maximum principal stress is given by

$$\sigma_{p1} = -\frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

This stress is not of much significance because it is reduced from higher magnitude σ or if it becomes tensile, to a magnitude which is still not much. However maximum shearing stress is significant.

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{4W}{2\pi d_1^2}\right)^2 + \left(\frac{16 M_t}{\pi d_1^3}\right)^2}$$

or $$\tau_{max} = \frac{16}{\pi}\sqrt{\frac{W^2}{64 d_1^4} + \frac{M_t^2}{d_1^6}} \qquad \ldots (8.15)$$

The permissible shearing stress will be known but solving Eq. (8.15) will be too difficult as you can see that it contains fourth and sixth power of d_1 and M_t is also the function of d_m (or d_1 and d). Therefore it is recommended to calculate d_1 from Eq. (8.14) and using this value of d_1, calculate τ_{max}. Then you have to see that the calculated value of τ_{max} is less than permissible value of shearing stress.

Determining Number of Threads

The screw may be as long as required by consideration of geometry of machine. For example a lead screw may be as long as the length of the lathe bed. But in all cases the load transfer between the screw and nut will require total load to be shared among the threads on nut, which is smaller in length than the screw. The number of threads is decided on the basis of the load carried by thread surface perpendicular to core cylinder as shown in Figure 8.12. All threads in contact will carry axial force of the screw through uniformly distributed pressure p_b, in a square thread, the width and depth of each is equal to t. The thread section is shown on left hand side of Figure 8.12 and on right hand side wherein one thread is shown loaded by pressure. The same pressure will be acting on the thread of the nut. Area on which pressure is acting is the area between the compressive circles of diameters d and d_1 for one thread and if there are n threads in contact or n threads on the nut, then total area of contact to carry the pressure.

$$A_p = n \pi d_m t = n \pi (d^2 - d_1^2)$$

$$\therefore \text{ Permissible pressure } p_b = \frac{4W}{n\pi (d^2 - d_1^2)} = \frac{W}{n \pi d_m t} \qquad \ldots (8.16)$$

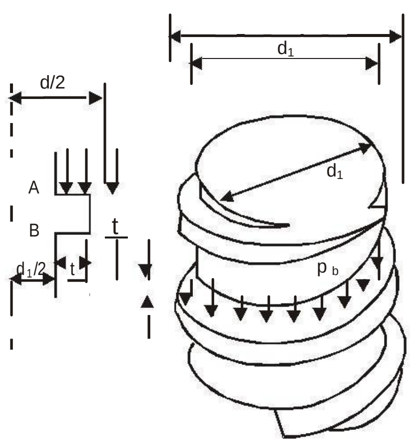


Figure 8.12 : Pressure on Contact Surface

This equation can be used for a check if the pressure between the threads of the nut and screw is within the permissible limit or it can be used to calculate the number of threads, n, in contact

Shearing of Threads from the Core Cylinder

Because of the force *W*, acting as uniformly distributed pressure as shown in Figure 5.12 thread may have a tendency to shear. The area over which shear effect will occur is shown shaded in Figure 5.13. The thread is shown in broken lines. This area apparently is a strip of width, *t*, on the cylinder of diameter d_1. Hence the area

$$A_c = \pi\, d_1\, t$$

With shearing stress τ, which will be created at the bottom of *n* thread of screw?

$$\tau_{screw} = \frac{W}{n\,\pi\, d_1\, t} \qquad \text{. . . (8.17)}$$

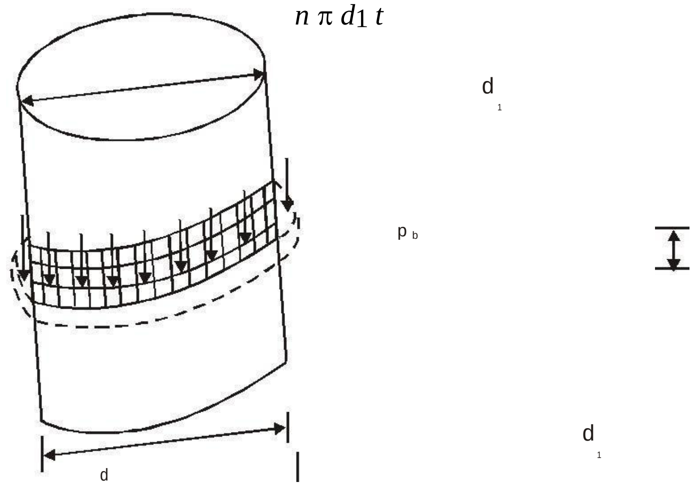


Figure 8.13 : The Area at the Bottom of a Thread in Screw

You must realize that the threads on the inside of the nut will also be similarly subjected to shearing stress at their bottom. Hence, in nut

$$\tau_{nut} = \frac{W}{n\,\pi\, d\, t} \qquad \text{. . . (8.18)}$$

Normally, the screw and nut are not made in the same material. While screw is made in steel the preferred material for nut is either cast iron or bronze. The permissible value of shearing stress for nut material may be less and in that case Eq. (8.18) must be used to calculate *n*.

The height of the nut is simply the product if *n* and *p*, i.e.

$$h = np \qquad \text{. . . (8.19)}$$

You must realise that the nut is threaded all along its length.

The various screw-nut material combinations are described in Table 5.2.

Table 8.2 : Screw-Nut Material Combination and Safe Bearing Pressure

Application	Material		Safe Bearing Pressure (MPa)	Rubbing Velocity at Mean Diameter m/min
	Screw	**Nut**		
Hand Press	Steel	Bronze	17.5-24.5	Well lubricated
	Steel	C.I	12.5-17.5	Low Velocity
Screw Jack	Steel	C.I	12.5-17.5	Velocity < 2.5
	Steel	Bronze	10.5-17.5	Velocity < 3.0
Hoisting Machine	Steel	C.I	4.0-7.0	6-12
	Steel	Bronze	35.0-100.0	6-12
Lead Screw	Steel	Bronze	10.5-17.0	> 15.0

Problem:

A screw press is required to exert a force of 50 kN when applied torque is 560 Nm. The unsupported length of the screw is 450 mm and a thrust bearing of hardened steel on cast iron is provided at the power end.

The permissible stresses in the steel screw are :

Tension and compression – 85 MPa, Shear – 55 MPa,

The permissible bearing pressure is 13.5 MPa for steel screw and C.I nut

The permissible shearing stress in the CI is 20 MPa

The yield strength of steel of screw, $\sigma_Y = 260$ MPa

The coefficient of friction in screw and nut is 0.15

Determine the dimensions of screw and nut, and efficiency.

Solution:

Step 1

Determine diameter d_1, d, p, α to define the thread

Eq. (8.14), will be used to estimate d_1,

$$\sigma = \frac{1.3W}{\frac{\pi d_1^2}{4}}$$

Use $W = 50{,}000$ N, $\sigma = 85$ N/mm^2

$$\therefore \qquad d_1^2 = \frac{4 \times 1.3 \times 50{,}000}{\pi \times 85} = 973.54$$

$\therefore \qquad d_1 = 31.2 \text{ mm}$

Look in the Table 8.1, close to 31.2 mm the core diameter is 33 mm with pitch $p = 7$ mm.

However, you may also use following rule : between

$d_1 = 30$ and $d_1 = 40$ mm, $p = 0.2\, d_1$. We choose to use Table 8.1.

So $\quad d_1 = 33$ mm, $d = 40$ mm, $p = 7$ mm . . . (i)

$$d_m = \frac{d_1 + p}{2} = 33 + 3.5 = 36.5 \text{ mm}$$

Angle of helix, $\alpha = \tan^{-1} \dfrac{p}{\pi d_m} = \tan^{-1} \dfrac{7}{\pi \times 36.5} = \tan^{-1} 0.061$

$$\alpha = 3.5^{o} \qquad \text{. . . (ii)}$$

$$\tan \phi = \mu = 0.15, \phi = \tan^{-1} 0.15$$

$$\phi = 8.53^{o} \qquad \text{. . . (iii)}$$

Efficiency of screw, $\eta = \dfrac{\tan \alpha}{\tan (\alpha + \phi)} = \dfrac{\tan 3.5}{\tan (3.5 + 8.53)} = \dfrac{0.061}{0.213}$

$$\eta = 28.6\% \qquad \text{. . . (iv)}$$

Number of threads in contact, i.e. threads in nut and height of nut.

Use Eq. (8.18), put $t = \dfrac{p}{2} = 3.5$ mm, $\tau_{nut} = 20$ N/mm^2

$$\tau_{nut} = \frac{W}{n \pi d t}$$

$$20 = \frac{50,000}{n \pi \times 40 \times 3.5}$$

$$n = 5.684 \text{ say } 6$$

This has to be checked for bearing pressure, $p_b = 13.5$ N/mm^2 Use Eq. (8.16)

$$p_b = \frac{W}{n \pi (d^2 - d_1^2)}$$

$$\therefore \qquad n = \frac{4 \times 50,000}{\pi \times 13.5 (40^2 - 33^2)} = 9.23 \text{ say } 10$$

This n is greater than the earlier calculated 6. Hence $n = 10$ is chosen.

Check for maximum shearing stress

$$M_t = W \tan (\alpha + \phi) \frac{d_m}{}$$

Use $$d_m = \frac{d + d_1}{2} = \frac{40 + 33}{2} = 36.5\,\text{mm}$$

$$M_t = 50{,}000 \times \tan(3.5 + 8.53) \times \frac{36.5}{2} = 19.4 \times 10^4 \text{ Nmm}$$

$$\tau = \frac{16\, M_t}{\pi\, d_1^{\,3}} = \frac{16 \times 19.4 \times 10^4}{\pi \times (33)^3} = \frac{98.8 \times 10^4}{3.594 \times 10^4}$$

or $\tau = 27.4 \text{ N/mm}^2$

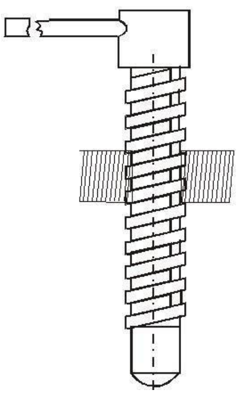
Also note $\sigma = \dfrac{4W}{\pi d_1^2} = \dfrac{4 \times 50{,}000}{\pi \times (33)^2} = 58.5 \text{ N/mm}^2$

$$\therefore \quad \tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \sqrt{855.56 + 750.76} = \sqrt{1606.32}$$

$$\therefore \quad \tau_{max} = 40.1 \text{ N/mm}^2$$

The permissible shearing stress is 55 N/mm^2.



Hence screw in safe against shearing.

Problem:

Design a screw jack to lift a load of 100 kN through a height of 300 mm. Assume σ_u = 400 MPa, τ_u = 200 MPa, σ_Y = 300 MPa, p_b = 10 MPa. The outer diameter of bearing surface is 1.6 d_1 and inner diameter of bearing surface is 0.8 d_1.

Coefficient of friction between collar on screw and C.I is 0.2. Coefficient of friction between steel screw and bronze nut is 0.15. Take a factor of safety of 5 for screw and nut but take a factor of safety of 4 for operating lever.

Solution:

The screw jack to be designed is shown in Figures 8.15(a) and (b) shows the details of the cup on which the load W is to be carried.

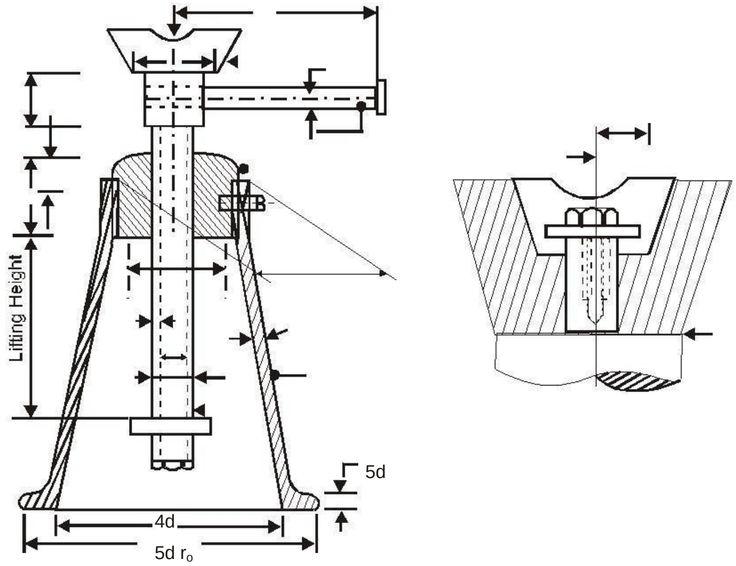


Figure 8.14 Screw jack assembled

Figure 8.15 : (a) Screw Jack Assembled; and (b) The Load Cup

The overall design of screw jack comprises designing of

(a) Screw,

(b) Nut,

(c) Arm,

(d) Cup, and

(e) Body of the jack.

Screw

Screw becomes the central part. The other parts will be dependent upon the screw. The screw design will decide core diameter d_1, major diameter d and pitch, p. Hence screw design will consist of calculating d_1, d and p and checking for maximum shearing stress and buckling of the screw.

Assume square thread

Use Eq. (5.14) with $W = 100{,}000$ N, $\sigma = \frac{400}{5} = 80$ N/mm^2

$$\sigma = \frac{1.3W}{\frac{\pi d_1^2}{4}}$$

$$\therefore \qquad d_1 = \left(\frac{5.2 \times 100,000}{\pi \times 80} \right)^{2} = (2069)^{2}$$

or $d_1 = 45.5$ mm

From Table 5.1, choose next higher value as $d_1 = 46$ mm with p = 9 mm and d = 55 mm.

After estimated values we go to check for maximum shearing stress and the buckling of the screw.

Maximum Shearing Stress

Compressive stress,

$$\sigma = \frac{4W}{\pi d_1^2} = \frac{4 \times 10^5}{\pi \times (46)^2} = 60.17 \text{ N/mm}^2$$

Torque on the screw, $M_t = W \tan(\alpha + \phi) \frac{d_m}{2}$

$$d_m = \frac{d + d_1}{2} = \frac{55 + 46}{2} = 50.5 \text{ mm}$$

$$\alpha = \tan^{-1} \frac{p}{\pi d_m} = \tan^{-1} \frac{9}{\pi \times 50.5} = \tan^{-1} 0.0567 = 3.25^o$$

$$\phi = \tan^{-1} \mu = \tan^{-1} 0.15 = 8.53^o$$

$$M_t = 10^5 \tan(3.25 + 8.53) \times \frac{50.5}{2} = \frac{0.21 \times 50.5 \times 10^5}{2}$$

$$= 5.266 \times 10^5 \text{ N-mm}$$

$$\tau = \frac{16 M_t}{\pi d_1^3} = \frac{16 \times 5.266 \times 10^5}{\pi \times (46)^3} = 27.6 \text{ N/mm}^2$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{60.17}{2}\right)^2 + (27.6)^2} = \sqrt{1666.87} = 40.827 \text{ N/mm}^2$$

The permissible sharing stress is $= \frac{\tau_u}{5} = \frac{220}{5} = 44$ N/mm .

Thus the screw is safe against shear.

Nut

For standard square thread the depth or thickness of the thread,

$$t = \frac{p}{2} = \frac{9}{2} = 4.5 \text{ mm}$$

2 2

Use Eq. (8.16), $p_b = \dfrac{W}{n \pi d_m t}$ with p_b = 10 N/mm^2, d_m = 50.5 mm,

$t = 4.5$ mm, $W = 10^5$ N

$$\therefore \quad n = \frac{10^5}{10\pi \times 50.5 \times 4.5} = 14$$

Normally $n > 10$ is not preferred. So we can go for higher d_1, d and p. Next standard values will be $d_1 = 49$ mm, $d = 58$ mm, $p = 9$ mm.

Hence, $d_m = \frac{58 + 49}{2} = \frac{107}{2} = 53.5$ mm, $t = \frac{p}{2} = 4.5$ mm

$$n = \frac{10^5}{10\pi \times 53.5 \times 4.5} = 13.2$$

Since this is also greater than 10, we can go for next higher value of $d_1 = 51$ mm with $d = 60$ mm and $p = 9$ mm.

So that $\quad d_m = \frac{51 + 60}{2} = \frac{111}{2} = 55.5$ mm

Which will also not satisfy the condition?

The last choice in the table with same p is $d_1 = 53$ mm and $d = 62$ mm.

So that $d_m = \frac{53 + 62}{2} = \frac{115}{2} = 57.5$ mm which result in $n = 12.3$ mm.

Still better solution is to go for next series with $p = 10$ mm.

With $d_1 = 55$, $d = 65$ mm, and $d_m = \frac{120}{2} = 60$; $t = 5$ mm

$$\therefore \quad n = \frac{10^5}{10\pi \times 60 \times 5} = 10.6$$

Which is very close to 10, hence can be accepted.

So the solution changes to $d_1 = 55$ mm, $d = 65$ mm, $p = 10$ mm. You need not check these dimensions because they are larger than the safe ones.

(It is important that the reader understands the reiterative nature of design and how the help from standards is derived. The iterations have been done to emphasize that the exercise in design should not be treated as a problem in strength of materials. In design the problem serves to bring practicability in focus.

The length of the nut, $H = np = 10.6 \times 10 = 106$ mm. Outside Diameter of Nut; see Figure

5.1(b) and Figure 5.9. The nut is in tension. The section to bear tensile stress is

$$\frac{\pi}{4}(D^2 - D_0^2)$$

$D = d + 0.5$ mm $= 65 + 0.5 = 65.5$ mm

Bronze is not as strong as steel. Silicon bronze (Cu = 95%, Si = 4%, Mn = 1%) is quite good for making nut. This material is available in wrought condition with $\sigma_u = 330$ MPa.

If factor of safety of 5 is used, then permissible tensile stress is $= 66\ \text{N/mm}^2$.

$$\therefore \qquad \sigma_t = \frac{4W}{\pi(D_0^2 - D^2)} = 66 = \frac{4 \times 10^5}{\pi(D_0^2 - 65.5^2)}$$

$$\therefore \qquad D_0^2 = 1929 + 4290 = 6219$$

or $\qquad D_0 = 78.86$ say 79 mm

To fit the nut in position in the body of jack, a collar is provided at the top (see Figure 8.15(a)).

The thickness of the collar $= 0.5\ D = 0.5 \times 65.5 = 32.72$ mm

The outside diameter of the Collar D_{00} can be found by considering crushing of collar surface under compressive stress. The area under compression is

$$= \frac{\pi}{4}(D^2 - D^2).$$

The compressive strength of bronze is same as tensile stress, 66 MPa

$$66 = \frac{4W}{\pi(D_{00}^2 - 79^2)} = \frac{4 \times 10^5}{\pi(D_{00}^2 - 6241)}$$

$$D_{00} = \sqrt{1929 + 6241} = \sqrt{8170} = 90.4 \text{ mm}$$

The nut is shown in Figure 8.16.

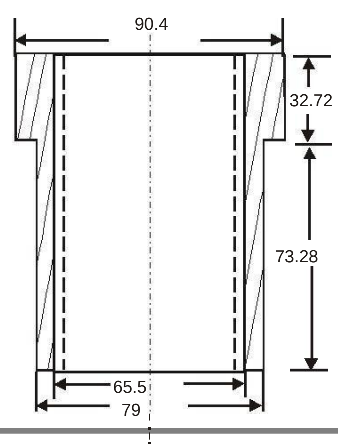


Figure 5.16

Arm

The arm is used to rotate the screw in the stationary nut. The portion of the screw at the top is enlarged to a diameter, $2r_0$

where $2r_0 = 1.6\, d_1 = 1.6 \times 55 = 88$ mm (see Figure 8.15(b)).

$2r_0$ is the outer diameter for bearing surface of collar on which will rest the load cup. The cup will turn around a pin of dia. $2r_i = 0.8\, d_1$,

$2r_1 = 0.8 \times 55 = 44$ mm.

These two diameters will be used to calculate the torque of friction at bearing surface. Call this torque M_{tf} and use Eq. (8.13)

$$M_{tf} = \frac{\mu_c W}{3} \frac{(2r_0)^3 - (2r_i)^3}{(2r_0)^2 - (2r_i)^2}$$

$\mu_c = 0.2$ and all other values are described earlier

$$M_{tf} = \frac{0.2 \times 10^5}{3} \frac{(88)^3 - (44)^3}{(88)^2 - (44)^2} = \frac{0.2 \times 10^5 \times (44)}{3} \times \frac{8 - 1}{4 - 1}$$

$$= 6.84 \times 10^5 \text{ Nmm}$$

The arm will have to apply this torque along with the torque for lifting the load which is given as : $M_t = W \tan(\alpha + \phi) \frac{d_m}{2}$

$$\phi = 8.53^\circ,\ d_m = 60 \text{ mm},\ \alpha = \tan^{-1} \frac{p}{\pi d_m} = \tan^{-1} \frac{10}{\pi \times 60} = \tan^{-1} 0.05 = 3^\circ$$

$$M_t = 10^5 \times 30 \tan(11.53) = 6.12 \times 10^5 \text{ N-mm}$$

Total torque, $M = M_{tf} + M_t = (6.84 + 6.12) \times 10^5 = 13 \times 10^5$ N-mm

A single man can apply 400 N of force but two men can apply 80 N with an efficiency of 90%. Let‟s assume two persons work at the end of the arm, the length of the arm

$$L = \frac{13 \times 10^5}{0.9 \times 800} = 1805.5 \text{ mm or } 1.805 \text{ m}$$

Actual length will incorporate allowance for grip and insertion in collar so arm of 2.1 m length will be appropriate.

The torque M will acts as bending moment on the arm with permissible bending stress as 160 N/mm^2. The diameter of arm

$$d_a = \left(\frac{32\,M}{\pi \sigma_b} \right)^{\frac{1}{3}} = \left(\frac{32 \times 13 \times 10^5}{\pi \times 160} \right)^{\frac{1}{3}} = 43.75 \text{ mm}$$

The Cup

The shape of the cup is shown in Figure 8.15(b). It can be made in C1 and side may incline 30^o with vertical. With bottom diameter as 1.2 (2r_i), its height can be decided by the geometry of load to be lifted. Since no information is given we leave this design only at shape.

The Body of the Jack

Height = Lifting height +length of the nut – length of the collar on the nut + allowance for bottom plate of 2 mm thickness and head of the bolt holding plate.

= 300 + 106 – 32.72 + (2 + 4.8) (using a 6 mm bolt)

= 380.1 mm

Thickness, $\delta = 0.25\, d = 0.25 \times 65 = 16.25$ mm

Thickness of base, $\delta_1 = 0.5\, d = 0.5 \times 65 = 32.5$ mm

Diameter of base, $D_2 = 4.0\, d = 4 \times 65 = 260$ mm

Diameter of base (outside), $D_3 = 5d = 5 \times 65 = 325$ mm .

Efficiency

$$\eta = \frac{\tan \alpha}{\tan (\alpha + \phi)} = \frac{0.05}{0.204} = 24.5\%$$

Listing of Designed Dimensions.

Screw : Square Thread.

Core diameter of screw, d_1 = 55 mm

Major diameter of screw, d = 65 mm

Pitch of Thread, p = 10 mm.

Nut

Number of threads, $n = 10.6$

Height of the nut, $H = 106$ mm

Outside diameter $D_0 = 79$ mm

Diameter of Nut collar, $D_{00} = 90.4$ mm

Thickness of collar = 32.72 mm

Arm of the Jack

The length of the arm = 2.1 m

The diameter of the arm = 43.57 mm

The Body of the Jack

Thickness of the body, $\delta = 16.25$ mm

Thickness of the base, $\delta_1 = 32.5$ mm

Inside diameter of base, $D_2 = 260$ mm

Outside diameter of base, $D_3 = 325$ mm

Height of the body $H_1 = 380.1$ mm